The Flow of Mechanical Energy in Convective Boundary Layers*

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Abstract

There are two frameworks within which we can discuss turbulence energy in convective boundary layers. The first is the one provided by the Reynolds-averaged Navier Stokes (RANS) energy equations, as interpreted by Osborne Reynolds in the late nineteenth century. The other, much newer framework is that provided by complex dynamical systems. The former gives prominence to the interpretation of local budgets of turbulence kinetic energy while the latter emphasizes the energy flows necessary to maintain turbulence in a statistically-steady state. It is argued that these frameworks constitute two incompatible paradigms, since the first localizes physical interpretation of the RANS kinetic energy budget while the second denies such a simple view. The local interpretation traces back to the way Reynolds himself interpreted his RANS energy equations, which interpretation is examined here and found to be faulty. We present a schematic model for energy flow in convective boundary layers from a complex dynamical systems' perspective, and use it to re-interpret the RANS energy equation.

1 Introduction

In his book *Hydrodynamics*, Lamb (1916) wrote that turbulence is "*the chief outstanding difficulty of our subject*". This remains true today, almost a century later. We know the governing equations but we can't solve them except by numerical integration, and even that remains impractical for flows with high Reynolds or Rayleigh numbers. Practical calculations of flows such as convective boundary layers (CBLs) therefore rely on models that combine empirical information, physical reasoning and intuition, all constrained by whatever guidance can be obtained directly from the Navier-Stokes equations. That is, the models rely on our *understanding* of each kind of turbulent flow, imperfect though that understanding may be. Here we compare two ways of understanding turbulence energy in CBLs.

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In the absence of rigorous theory, approaches to turbulence can be divided into two broad paradigms: statistical fluid mechanics (SFM) and complex dynamical systems (CDS). The first of these takes a fundamentally stochastic view of turbulence, albeit with an interpretative overlay that often crosses over with the dynamics. SFM has the longer history, with origins in the work of Osborne Reynolds in the late nineteenth century, and with major development by Andrey Kolmogorov and his Russian school in the 1940s. It includes the so-called Reynolds-averaged Navier-Stokes (RANS) equations, which equations have been used successfully to model many kinds of flow, albeit with a variety of closure assumptions whose success is tempered by a disconcerting lack of generality so that methods must be specially tailored for each narrow class of flow.

The second paradigm, CDS, takes a fundamentally deterministic, albeit chaotic, view of turbulence, and it has a shorter history with several strands. One strand originated in the 1950s with the contributions of Townsend (1956) on the forms of eddies; another with the work of Lorenz (1963) on chaotic flows; and a third—the non-equilibrium thermodynamics—with the work of Paltridge (1975) on the maximum entropy production principle in the context of the global climate system. This principle now has solid theoretical support (Dewar, 2003) but has yet to receive significant attention in boundary-layer studies (but see Wang (2009) for an early attempt, albeit one that mixes its paradigms). What draws these strands together is the idea that CDSs are self-organizing, pattern-forming systems, and that turbulent flows, where the recurring patterns may be called ‘coherent structures’ or, more-generally, ‘eddies’, may be understood as a highly-organized system of interacting eddies. A high level of organization has its cost, however, since the organization will decay away unless it is supported by a continuous flow of energy. The CDS paradigm has yet to produce any practical applications.

The word *paradigm* is used carefully here, and to mean much what Kuhn (1970) would have it mean. That is, different paradigms imply not just different models, which may or may not be complementary, but different ways of understanding based on different fundamental concepts; these giving rise to different kinds of questions and different kinds of acceptable answers. Even the meaning of language can change between paradigms, so that dialogue can become difficult between the adherents of different paradigms. In this context we ask ‘What is *shear production* of turbulence kinetic energy?’ Is it, as the adherents of SFM would have it, the local passing of kinetic energy from the mean flow to the fluctuations; or could ‘production’ of energy apply only to the introduction of energy into a system from sources external to that system—all else being simply the moving of energy around within the system and not ‘production’ at all—as might be maintained from a CDS standpoint.

The SFM paradigm has lead us to search for a local, layer-by-layer understanding of boundary-layer processes, based on differential equations linking ensemble-averaged statistical quantities, with local turbulence energy budgets as a key component. The CDS paradigm, on the other hand, says that the governing differential equations, which are inherently local relationships, do not have the power to interpret flow processes until integrated, since the essential patterns of motion—the energy-bearing structures—appear only when the governing differential equations are integrated, either digitally or in nature. The CDS paradigm is that flow processes are ultimately non-local and arise as a consequence of the emergent patterns of motion (eddies), which are properties of the whole flow. Thus the terms of the RANS energy budget equation cannot simply be interpreted as the energy production caused by the local action of the forces represented in the Navier-Stokes equations. In CDS, causality within a system necessarily occurs in circular chains powered by energy flowing from without the system, just as chickens and eggs occur in cycles powered by energy supplied as food. Thus the smaller-scale eddies and thermal structures combine to constitute and drive the larger ones, and the larger-scale eddies and thermal structures carry the smaller ones along within them, and so organize and drive the smaller ones. The SFM and CDS paradigms also lead to different modelling constructs. For example, SFM modelling employs a 'surface layer' in the region of a CBL adjacent to the ground while CDS leads to a differently-defined 'surface friction layer' (SFL). The surface layer is the layer where local values of fluxes are sufficiently close to their values at the ground that the latter can be used in local modelling throughout the layer. The surface friction layer, by contrast, is defined in terms of the kinds of eddies found there. In practical terms, SFM has lead to the development of Monin-Obukhov similarity theory (Monin and Obukhov, 1954) that applies to the statistical properties of the whole flow at each point, while CDS leads to scaling (similarity) models that are specific to each class of eddies (McNaughton et al. 2007; Laubach and McNaughton, 2009).

Paradigms ultimately survive or fail depending on how well their constructs and relationships correspond to those found in the real world, and on how well they lead to the asking of useful new questions. Internal contradictions can destroy a paradigm, but often not immediately since a new paradigm might be hard to find and, when found, be markedly less well developed and so less useful in the short term. It is the opinion of the present author that boundary-layer meteorology is now entering a phase where the shortcomings of the SFM paradigm, and the advantages and opportunities entailed in the CDS paradigm, are becoming apparent. The present discussion of energy production and flow in CBLs is intended to highlight this.

2 Energy Flows in the CBL

We begin by taking a CDS perspective and look at the energy flows in a horizontally-homogeneous CBL. Energy flows are a central concern because energy must flow through any CDS if its patterns are to be maintained against the universal tendency to disorder.

The energy flows in a CBL are shown schematically in Fig. 1, based on the description by Laubach and McNaughton (2009). Energy sources are indicated by the three boxes on the left, which break the outline of the main box enclosing the CBL system. The shear eddies shown in the box at the bottom of Fig. 1 occur within the SFL. This layer usually

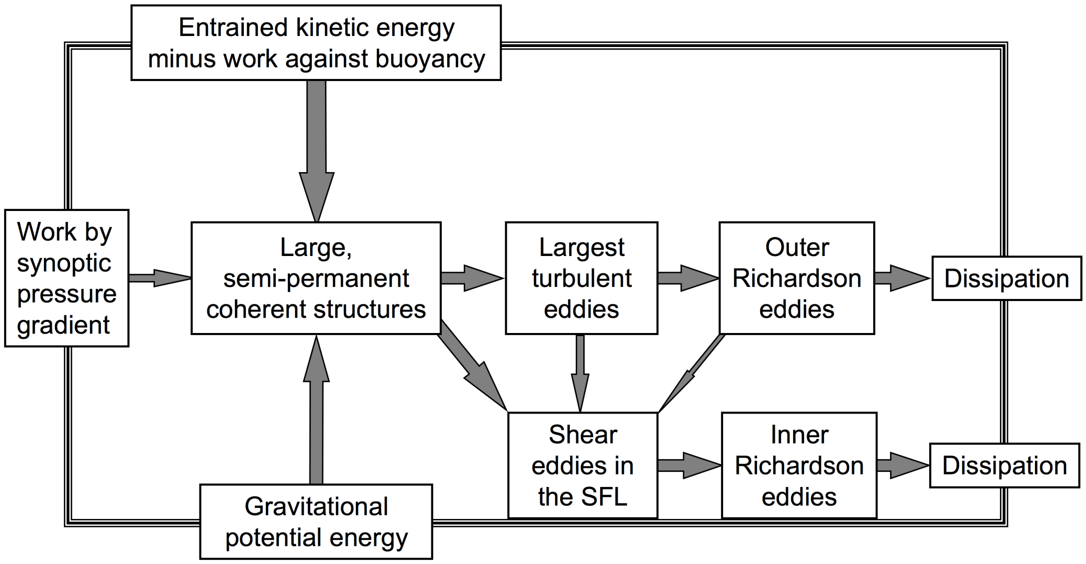

Figure 1: The flow of mechanical energy in a growing convective boundary layer (CBL), based on the structural model of Laubach and McNaughton (2009). The CBL is a thermodynamically open system, with external energy sources indicated by the three boxes on the left of the diagram that break the outline of the main system box. The top of the CBL system, though drawn schematically as a straight line, is the convoluted surface separating the air within the CBL, characterized by small, dissipating eddies, from the quieter air above. The arrows indicate energy flows from one kind of structure to another. In each case the transfer is from larger and simpler structures to smaller and less organized structures. The large, semi-permanent coherent structures are the horizontal roll vortices or cellular structures that comprise the mean wind. Instabilities in these structures lead to the formation of large, z_i-scale eddies, with energy flowing first into these and then on, down the outer Richardson cascade until it is passed to eddies small enough to dissipate it as heat, and so out of the system. The large coherent structures, the z_i-scale eddies and the largest of the outer Richardson eddies create mean and variable shear at the ground. This creates a surface friction layer (SFL), of height z_s, characterized by its own peculiar shear eddies. These eddies break up in their turn, and they pass their energy on down the inner Richardson cascade to dissipation. In light winds and with a smooth surface the SFL height, z_s, might be less than 1 metre, while with strong winds and over a rough surface the SFL may reach up to the top boundary of the system, at z_i. We might call this diagram the ‘trophic diagram’ for energy flow in a CBL, by analogy with similar diagrams for food chains in ecosystems.

displays substantial wind shear and higher levels of turbulence kinetic energy (TKE) than found in the rest of the CBL. By common consensus, it is this wind shear and proximity to the ground that gives rise to the intense eddying and high rates of dissipation found there. The flow schematic is partly geographically organized, making it clear that most of the turbulence energy dissipated within the SFL flows down from above and is not 'produced' within the SFL.

To demonstrate this we begin with an expression for the vertical advection of kinetic energy, $w(u^2 + v^2 + w^2)/2$, following Taylor (1952), where (u, v, w) are the components of the velocity field, then perform a Reynolds' expansion by expressing each component as a sum of a mean and a fluctuating part,

$$u = \overline{u} + u' \quad (1)$$

$$v = \overline{v} + v' \quad (2)$$

$$w = \overline{w} + w' \quad (3)$$

and then apply the Reynolds averaging rules to obtain an expression for the vertical advection of total kinetic energy. Next, we take its vertical divergence, and lastly ensemble-average the whole expression to obtain an expression for the averaged vertical flux of kinetic energy, so

$$\frac{1}{2}\,\frac{\partial\,\overline{w(u^2 + v^2 + w^2)}}{\partial z} = \overline{u'w'}\,\frac{\partial \overline{u}}{\partial z} + \overline{u}\,\frac{\partial \overline{u'w'}}{\partial z} + \frac{\partial \overline{w'e}}{\partial z} \quad (4)$$

where e is the turbulence kinetic energy (TKE), $e = (u'^2 + v'^2 + w'^2)/2$, and we have set $\overline{v} = \overline{w} = 0$, as befits a plane, horizontally-homogeneous flow in a CBL.

Near the ground, at heights below $-0.5L$ where L is the Obukhov length, the first term on the right is negative because $\overline{u'w'}$ is negative and $\partial\,\overline{u}/\partial z$ positive. Experiments show that this term is the largest of the terms on the right of (4); the second term is small because $\overline{u'w'}$ is approximately constant with height; and the third term is usually small compared to the first (e.g. Li et al. 2008). Thus the vertical divergence of the downwards flux of kinetic energy is negative near the ground, contributing energy to turbulence processes at each level. Figure 1 illustrates this, showing that entrainment provides the ultimate source of most of the energy flowing into the surface friction layer (except in near windless conditions). Only a very small part of the energy is derived from work done by the imposed pressure gradient. Eq. 4 is therefore consistent with Fig. 1.

Let us take stock of what has just been done. Equation 4 is a formally correct statement, based on the definitions of kinetic energy and of the ensemble means of the velocity components, as introduced by Kolmogorov and his Russian school (Monin and Yaglom, 1971). The argument starts with an expression, $w(u^2 + v^2 + w^2)2$, whose meaning we can understand at each instant and at each point in a single realization of the flow, so we have a reliable physical interpretation of its ensemble mean, and of the divergence of that ensemble mean, on the left-hand side of (4). The manipulations used to develop the terms on the right employ Reynolds' expansion and averaging rules, which are themselves formally correct when the means are defined as ensemble means. These manipulations entail no new physics—not even that implicit in the Navier-Stokes equations. Equation 4 is therefore purely an accounting equation, necessarily correct for a horizontally-homogeneous flow, but implying no particular interpretations of the individual terms on the right. Identification of $\overline{u'w'}\,\partial\overline{u}/\partial z$ as the largest term on the right-hand side of (4) relies on empirical information, but no new physics. The term $\overline{u'w'}\,\partial\overline{u}/\partial z$ is the largest part of the divergence of the downwards flux of kinetic energy near the ground. This interpretation should be as correct within the CDS paradigm as it is within the SFM paradigm, since both paradigms respect the primacy of experimental evidence. We might say that there could be no possible argument with this interpretation, but we would be wrong since all the major textbooks (Lumley and Panofsky, 1964; Monin and Yaglom, 1971; Wyngaard, 2010) have a different interpretation, and that interpretation lies within the SFM paradigm.

3 Shear Production of Turbulence Kinetic Energy

The textbooks all interpret $-\overline{u'w'}\,\partial\overline{u}/\partial z$ as the local transfer of kinetic energy from the mean flow to the turbulent fluctuations, and they describe this process as *shear production* of TKE. Their argument has two parts: the first is a formal development to produce the Reynolds-averaged Navier-Stokes (RANS) equations for the budgets of mean and turbulent kinetic energy, and the second part is an interpretation based on physical arguments. Both parts trace back to the work of Reynolds (1895), though Reynolds employed volume rather than ensemble averaging so that rigour was not achieved in his formal development.

The modern textbooks all use ensemble averaging, so their derivations are formally correct. Beginning with the Navier-Stokes equations and making substitutions using the Reynolds decomposition of field variables into mean and fluctuating parts, then manipulating the results with the aid of Reynolds' rules for the behaviours of the averaged variables, they obtain equations for the kinetic energy of the mean and fluctuating parts of the flow. For a horizontally-homogeneous flow these can be represented as

$$\frac{\partial E}{\partial t} = \ldots + \overline{u'w'}\,\frac{\partial \overline{u}}{\partial z}, \tag{5}$$

and

$$\frac{\partial e}{\partial t} = \ldots - \overline{u'w'}\,\frac{\partial \overline{u}}{\partial z} \tag{6}$$

respectively. In these equations "..." represents terms that differ between the two equations. The term $\overline{u'w'}\,\partial\overline{u}/\partial z$ appears on the right-hand side of both (5) and (6), being added in (5) and subtracted in (6). It comes from the expansion of the non-linear advection terms in the Navier-Stokes equation, and signifies a formal coupling of the two equations, so that neither can be solved independent of the other. So much is achieved purely by definition and formal manipulation, so (5) and (6) are unexceptionable as accounting equations. Accurate measurements of the various terms will satisfy them exactly when averaged over a sufficient number of flow realizations.

The textbooks, however, go further and interpret these terms as representing a real, local transfer of kinetic energy from the mean part of the flow to the fluctuating part. This explanation has become part of the SFM paradigm since the textbooks do not distinguish between the formal aspects of (5) and (6) and their physical interpretation. But this creates a problem because we now have two interpretations of $-\overline{u'w'}\ \partial\overline{u}/\partial z$, one from (4) and the other from (5) and (6), and the first of these is, as said above, unexceptionable. The second interpretation, that the TKE is created locally rather than transported in from somewhere else, must be wrong. The error must lie in the interpretation since the equations themselves are rigorously derived.

A careful reading of the textbooks shows that the arguments used in favour of the *shear-production* interpretation of $-\overline{u'w'}\ \partial\overline{u}/\partial z$ trace back to the work of Reynolds (1895) himself, so we must revisit his writings if we are to understand the origins of the standard SFM interpretation.

3.1 Reynolds' Paper of 1895

Reynolds' paper (Reynolds, 1895) was not given an easy passage by its editor and reviewers. We know this because the editorial correspondence on it has now been made available by the Royal Society of London (Jackson and Launder, 2007). Lord Rayleigh was then editor of the *Philosophcal Transactions of the Royal Society*, and he selected Sir George Stokes—he of the eponymous Navier-Stokes equations—as reviewer and, for a second opinion, Horace (later Sir Horace) Lamb, a mathematical physicist and author of the highly-respected book *Hydrodynamics* (Lamb, 1916). It would be hard to imagine three scientists better qualified to evaluate a paper of this nature. Unfortunately, even they found Reynolds' writing to be obscure and they could not understand his arguments. Rayleigh, acting on the recommendation of his reviewers, decided to accept the paper for publication, though he based this decision on Reynolds' own high standing rather than on any endorsement of his arguments.

Two things puzzled the reviewers. One was Reynolds' pernickety discussion of averaging methods and his insistence that they limited the generality of his conclusions to flows with linear mean velocity gradients, or to steady flows. The other was the substantial space that Reynolds devoted to discussing the molecular model of fluids and how the kinetic energy of eddies is converted into heat energy, which Reynolds saw as a central issue.

In a section of his paper, apparently added in response to the reviewers' comments, Reynolds re-asserted the importance of these matters, and wrote: "*My object in this paper is to show that the theoretical existence of an inferior limit to the criterion follows from the equations of motion as a consequence*
(1) Of a more rigorous examination and definition of the geometrical basis on which the analytical method of distinguishing between molar-motions and heat-motions in the kinetic

theory of matter is founded; and
(2) Of the application of the same method of analysis, thus, definitely founded, to distinguish between mean-molar-motions and relative-molar-motions where, as in the case of steady-mean-flow along a pipe, the more rigorous definition of the geometrical basis shows the method to be strictly applicable, and in other cases where it is approximately applicable."[R5], where the notation [Rx], here and elsewhere, indicates that the source is the article numbered 'x' in Reynolds' paper.

Though Reynolds' writing style is characteristically sinuous and obscure, we understand that he was looking for an explanation of why a laminar flow becomes turbulent at a particular value of his [Reynolds] number. He sought this explanation in the conditions necessary for energy to pass from the mean flow to the turbulent fluctuations at each point in the flow, and to do this Reynolds needed a reliable method for distinguishing between the mean flow and the fluctuations. This is no problem at integral scale, where the mean flow is simply the volume of water passing through the pipe per unit of area and time, but Reynolds needed to define local values of the mean velocity. He was familiar with the kinetic model of fluids, having made important contributions to this subject in his own work on the escape of molecules through surfaces having surface tension, so his search was guided by the known method for distinguishing between the molar (i.e. bulk) motions and the molecular (heat) motions of a fluid. To follow his argument we begin with Reynolds' ideas on molar and molecular motions.

3.1.1 Molar and Molecular Motions

Reynolds understood that energy is passed down from larger to smaller eddies and that this energy is eventually transformed to heat energy, and it was the final step that interested him: from the kinetic energy associated with the smooth molar motion of the bulk flow into heat energy, which is the mean kinetic energy of the erratic motions of the molecules. He knew that the molar motion at any point could be calculated by summing the momentum of all the molecules in a small volume of fluid surrounding that point, provided the selected volume is large enough that erratic motions are smoothed out. By making successive integrations over neighbouring volumes, one could then define the molar velocity as a smooth function of position, $\vec{u}(\vec{x})$. In this understanding Reynolds was up with, or ahead of, other scientists of his time since the molecular nature of fluids was not generally acknowledged until 1905, when Einstein published his work on Brownian motion.

That is not to say that Reynolds possessed a modern knowledge of small-scale processes in fluids. For example, he did not know that the smallest eddies have sizes comparable to the Kolmogorov microscale, and that this is always many orders of magnitude greater than the dimension of a suitable averaging volume. He therefore considered the case where the smallest eddies have sizes comparable to the diameter of the necessary averaging volume. The molar velocity field can then be curved within the volume needed to smooth the molecular motions. This means that the molar and molecular motions cannot be perfectly separated by the volume averaging method. However, Reynolds believed that a strict separation must be possible. This is a pivotal point in Reynolds' argument, so we quote him directly: "*The only known characteristic of heat-motions, besides that of being relative to the mean-motion, already mentioned, is that the motions of matter which result from heat are an ultimate form of motion which does not alter so long as the mean-motion is uniform over the space, and so long as no change of state occurs in the matter. In respect of this characteristic, heat-motions are, so far as we know, unique, and it would appear that heat-motions are distinguished from the mean-motions by some ultimate properties of matter.*" [R11]. Here Reynolds uses the term *mean-motion* to indicate a volume mean over the local molecular motions, while we use his alternative name, *molar motion*, for the same thing.

Since heat is an "*ultimate form of motion*" there must be an absolute distinction between molar and molecular motions, and so, argued Reynolds, there must be a mechanism able to maintain that distinction throughout the flow. He did not know what that mechanism might be, but Reynolds was certain that it must exist, and he called it the "*discriminative cause*" of scale separation. Reynolds then argued that, despite this absolute separation of scale, energy must still be transferred from the molar to the molecular motions, so there must be a physical mechanism able, at each point in the flow, to bypass the *discriminative cause* and so effect that transfer. He called that mechanism the "*cause of transformation*". Thus, for Reynolds, the molar and molecular motions were physically real and quite separate things, albeit things able to exchange kinetic energy by some unknown mechanism.

While Reynolds believed that molar and molecular motions are different *ultimate properties of matter*, we would say that they are both representations of the same underlying reality. Molecules (or something even more fundamental) are the stuff of all flows and the kinetic model, which represents a fluid as a collection of small, perfectly-elastic molecules, can be used to describe behaviour at scales comparable to the mean free path of the molecules. That an almost perfect separation of concepts of heat and molar motions can be achieved for gasses and liquids on earth is a consequence of the high velocities and small mean free paths of their molecules: suitable averaging volumes can be very small when compared with the dimensions of flow-measuring instruments; and the kinetic energy of bulk motions usually makes a wholly-negligible contribution to the mean kinetic energy of molecules. Reynolds could have developed these ideas himself in 1895 since velocities and mean free paths of molecules and the molecular basis of viscosity (a continuum property) had been known since the 1860s, following the work on the kinetic theory of gases by Rudolph Clausius and James Clerk Maxwell. Perhaps it suited Reynolds' purpose not to delve too far into these matters for he was next to apply his ideas to quite a different situation.

3.1.2 Mean and Eddy Motions

Believing that he had established that a *discriminative cause* is necessary to maintain the separate identities of molar and molecular motions, Reynolds argued for an analogous separation between the mean and fluctuating parts of the molar flow, which is to say between the mean flow and the eddies. He noted that, if the *discriminative cause* and *cause of transformation* depend on properties of matter that affect all modes of motion, distinctions in periods must exist between mean motions and fluctuations, and a transformation of energy must take place from the one to the other, as between the molar and molecular motions [R11]. He then argued that proof of the analogy should be sought in experiment [R12].

Reynolds' evidence for this analogy is presented in just one paragraph of his 42 page paper. He appeals to his earlier pipe experiments (Reynolds, 1983) and argues that energy must be passed from the mean flow to the fluctuations, since it is "*by which transformation the state of eddying-motion is maintained, notwithstanding the continual transformation of its energy into heat-motions*" [R13]. That is, Reynolds simply transferred his understanding of energy transformations in pipe flows at integral scale [R18] to the general case at local scale. In effect Reynolds offers no evidence at all for his act of localization.

Nevertheless, Reynolds was confident of his arguments and concluded: "*We have thus direct evidence that properties of matter which determine the cause of transformation, produce general and important effects which are not confined to the heat-motions.*"[R13]. In effect, Reynolds projected his belief that molar and molecular motions have distinct physical identities into the belief that mean molar motions and turbulent fluctuations have similar separate identities at each point in the flow. He also transferred from the molecular to the turbulent case his belief that there must exist a local *cause of transformation* to pass energy from the mean motion to the fluctuations.

Reynolds then went on to derive his famous RANS equations for the mean and fluctuating parts of the kinetic energy (his equations (17) and (19), summarized here as (5) and (6)). These are differential equations that express local energy budgets at each point in the flow. He noted the terms $\overline{u_i'u_j'}\,\partial\overline{u}_i/\partial x_j$ were added to the one and subtracted from the other, where we now use the Einstein summation convention for the general case. Reynolds identified this term with his *cause of transformation*, arguing: "*These terms which thus represent no change in the total energy of mean-motion can only represent a transformation from energy of mean-mean-motion to energy of the relative-mean-motions.*" [R16]. Reynolds' argument was complete to his own satisfaction, and his local interpretation of the RANS equations has largely gone unquestioned to the present day.

3.2 Reynolds' Legacy

It is clear that Reynolds' case was not proven. Indeed it was wrong: Reynolds' averaging methods are unable to separate mean and fluctuating parts of a flow in the way he had wanted. The largest eddies in statistically-steady turbulent flows (which is to say all flows with turbulence maintained by a steady flow of energy) typically span the turbulent part of that flow, so an averaging volume large enough to separate mean motion from the fluctuations would have to be as wide as the turbulent flow itself. Such a large averaging volume could define a single $\overline{u}$ averaged over the whole width of a turbulent flow, but the centre of gravity of the averaging volume could not then be moved around, and Reynolds' averaging method could not identify $\overline{u}(x, y, z)$, and with $\overline{u}(x, y, z)$ undefined for any single realization of the flow at any particular point and time, it is not possible to attribute a dynamical role to $\overline{u}(x, y, z)$.

Though the above comments are clearly critical of Reynolds' arguments, they are not critical of Reynolds' understanding that it is necessary to establish the physical reality and distinctness of the mean and fluctuating parts in a single realization of a flow if kinetic energy is to pass locally from the one part to the other. Unfortunately, and no doubt partly because of the obscurity of Reynolds writing, the importance of this point was not understood by his editor and reviewers or, it seems, by later researchers and textbook authors. By assigning dynamical roles to averaged quantities Reynolds transferred the local fluid dynamics applicable at each instant in a flow, as expressed directly by the Navier-Stokes equations, to a local, term-by-term interpretation of the RANS energy equation. This local interpretation was adopted directly, for example, by Richardson (1920), who added a *buoyant production* term to Reynolds' energy equation and so came to define his [Richardson] number as a measure of the local importance of buoyancy in the turbulence energy budget. This became a key part of the SFM paradigm when used in studies of the atmospheric boundary layer. Monin and Obukhov used Reynolds' method of local similarity analysis when developing their similarity theory for buoyant flows (Monin and Obukhov, 1954) and made a similar mistake.

Of course much has changed since 1895, and Reynolds' own averaging methods are not the ones described in modern textbooks. Ensemble means now replace Reynolds' volume means. This solves the problem of defining mean profiles $\overline{u}(z)$ in flows where the largest eddies span the flow, and it makes rigorous Reynolds' rules for dealing with averages. The RANS equations then become formally rigorous equations for accounting the statistics of flows. The terms with $\overline{u'w'}\, \partial\overline{u}/\partial z$ in Eqs. 5 and 6 remain as coupling terms, so that the one equation cannot be solved without the other. The new statistical interpretation does not rely on the smallness of eddies for its validity, and it is quite agnostic as to whether any real *physical* distinction can or should be made between the mean and fluctuating components of velocity. Any physical interpretation of the meanings of terms must come from somewhere else. In particular, such interpretations must be properly grounded on physical processes occurring in single realizations of the flow, since such single realizations are the proper domain of the Navier-Stokes equations. Reynolds attempted to do this, but failed because he wrongly assigned a real physical significance to the mean flow at each point in the flow. Despite this, Reynolds' interpretations survived the Russian statistical revolution, as formulated by Kolmogorov and his followers, and were simply adopted into the new SFM without further discussion. Reynolds' method of interpretation thus became an important part of the SFM paradigm. Indeed, it became the conceptual foundation for most of our models of turbulent flows throughout the 20th Century.

4 Interpreting the RANS Energy Equation

We have noted that the RANS energy equation is a formally-correct relationship between ensemble-averaged quantities, but argued that the SFM interpretation of that equation is wrong. Particular doubts attach to SFM interpretation of the the *shear production* term, $-\overline{u'w'}\,\partial\overline{u}/\partial z$. Here we examine the interpretation of that term according to CDS.

We begin by writing the RANS energy equation, Eq. 6, in full for a steady, horizontally-homogeneous CBL as

$$-\overline{u'w'}\,\frac{\partial\overline{u}}{\partial z}-\frac{\partial\overline{w'e}}{\partial z}-\frac{\partial\overline{w'p'}}{\partial z}+\frac{g\;\overline{w'T'}}{\overline{T}}-\epsilon=0 \qquad (7)$$

where g is the acceleration due to gravity, T is absolute temperature, p is pressure and ϵ is the local dissipation rate. On comparing this last equation with (4) we notice that the first and second terms on its left-hand side also occur in (4), so these terms may be eliminated by adding the two equations. This gives

$$\frac{\partial\,\overline{w(u^2+v^2+w^2)/2}}{\partial z}+\frac{\partial\overline{w'p'}}{\partial z}-\frac{g\;\overline{w'T'}}{\overline{T}}-\overline{u}\,\frac{\partial\overline{u'w'}}{\partial z}-\epsilon=0 \qquad (8)$$

This is an alternative statement of the energy balance at a particular level, but one in which we can identify each term as an ensemble average of physical processes operating at each instant and each point in each realization of the flow.

The first term of (8) is the ensemble average of the divergences in the vertical flux of kinetic energy in each realization. The second and third terms on the left-hand side are, respectively, the ensemble averages of the divergences in the fluxes of pressure potential energy and gravitational potential energy. This can be verified by first writing these quantities for each realization in terms of whole variables from first principles, then Reynolds-decomposing each variable and setting $\overline{w}=0$ and taking their ensemble averages. Thus the first three terms together comprise the vertical divergence in the flux of total mechanical energy. This must balance the gains and losses due to the actions of body forces and dissipation. The dissipation rate is the fifth term ϵ, leaving $-\overline{u}\,\partial\overline{u'w'}/\partial z$ to represent the work done by body forces.

The interpretation offered by Fig. 1 is that $-\overline{u}\;\partial\overline{u'w'}/\partial z$ is the work done by the synoptic pressure gradient, some of which must be done within the SFL. To see this we write the horizontal momentum balance at each level in the CBL as

$$-\vec{\hat{x}}\cdot\vec{\nabla}p=\frac{\partial uw}{\partial z} \qquad (9)$$

for any single realization of the flow, where $\vec{\hat{x}}$ is the unit vector in the streamwise direction. The synoptic pressure gradient is steady on all turbulent time scales so we can set $p' = 0$, and write the ensemble-averaged work done by this gradient as

$$-\overline{u \cdot \vec{\nabla} p} = -\overline{(\overline{u} + \vec{u'}) \cdot \nabla \overline{p}} = \overline{u}\ \frac{\partial \overline{u'w'}}{\partial z} \tag{10}$$

This confirms our identification of $-\overline{u}\ \partial\overline{u'w'}/\partial z$ as the work done at each level by the synoptic pressure gradient. No problems arise from any lack of a local physical interpretation of $\overline{u}(z)$ because no dynamical argument depends on it: the u fluctuations are simply eliminated by formal ensemble averaging when the synoptic pressure gradient is steady on all turbulence time scales.

With the physical meaning of $-\overline{u}\ \partial\overline{u'w'}/\partial z$ now clear we can interpret the remaining terms in (4), $\left(-\overline{u'w'}\ \partial\overline{u}/\partial z - \partial\overline{w'e}/\partial z\right)$, as representing the rate of gain of kinetic energy of the flow at each level caused jointly by the divergences of the three downwards fluxes of kinetic energy and the work done by the synoptic pressure gradient. Indeed, if z/z_i is not extremely small they may include a contribution from the buoyancy flux also. Whatever their composition, these residual terms occur as a pair and we may ask whether the two parts have any separate physical significances. This question is not a simple one since the terms together encompass several different flow pathways in Fig. 1, and which of these is important depends on the height at which we make our interpretation. We shall keep this in mind as we examine the RANS energy equation, Eq. 7.

4.1 Interpretation Within the SFL

We first choose an observation height located within the SFL, shown at the bottom of Fig 1. We must take care when using Fig. 1 to interpret Eqs. 7 and 8 because the arrows in Fig. 1 represent energy flows between different kinds of eddies, which may occur over a range of heights, while Eqs. 7 and 8 represent energy budgets at particular heights in the flow. Thus some of the divergence in the upwards flow of gravitational potential (buoyancy) energy will occur within the SFL; some of the work done by the synoptic pressure gradient will be done within the SFL; and some of the inner Richardson eddies may be carried up out of the SFL before dissipating at higher levels rather than where they are produced. The divergence in the flux of mechanical energy—the sum of the first three terms on the left-hand side of (8)—will then have five components: the divergences in the three energy flows shown feeding directly down into the SFL from the outer flow; the divergence in the upwards flux of gravitational potential energy; and the divergence in the energy associated with the upwards transport of Richardson eddies within the SFL.

Notice that Fig. 1 shows the gravitational potential energy passing directly from its source at the ground to the large, semi-permanent coherent structures (LCS), with none going directly to the much-smaller, momentum-transporting structures within the SFL. In this respect Fig. 1 is incompatible with the concepts developed by Richardson or Monin and Obukhov in their similarity models, both of which are based on the SFM paradigm. In SFM any warmer air parcel contributes kinetic energy to the overall TKE budget, without regard its size or form. Fig. 1 does have experimental support since it leads to a prediction that is verified experimentally. The LCS span the CBL so the divergence of the buoyancy flux contributes energy to them at all levels in the CBL, including levels within the SFL. Because the LCS are large their motions are essentially horizontal near the ground, so the vertical advection of kinetic energy must be small. The gravitational potential energy must then, by default, be transported by the pressure fluctuations. Noting that pressure transport is zero in neutral and stable flows, we can suppose pressure transport is not associated with the shear eddies found within the SFL, so all of the gravitational potential energy must be transported by pressure in the limit $z/z_i \to 0$. Therefore we have

$$\frac{\partial \overline{w'p'}}{\partial z} \approx \frac{g\ \overline{w'T'}}{\overline{T}} \tag{11}$$

for $z \ll z_i$. This relationship is confirmed by experimental studies of the RANS energy budget within the SFL (Wyngaard, 1992), which supports the scheme shown in Fig. 1. The same conclusion was reached by McNaughton (2006), using arguments based on the nature of the momentum-transporting eddies within the SFL.

There is also experimental evidence that some Richardson eddies survive long enough to be carried up out of the SFL within the plumes that transport heat and other scalars up out of the CBL (Khalsa, 1980; McNaughton, 2007). This would constitute a net energy flow upwards from the SFL, and the divergence in this flux at each level would be included in the first term of Eq. 8 as part of the divergence of the kinetic energy flux within the SFL. Thus some of the energy transferred down the inner Richardson cascade in Fig. 1 is dissipated within the SFL and some above it.

The remaining process to be considered is the down-scale transfer of energy from the larger structures in the CBL to the smaller, momentum-carrying turbulent structures within the SFL. In particular, we can now ask if we can associate the term $-\overline{u'w'}\ \partial\overline{u}/\partial z$ with the action of some kind of *mean flow* in the CBL, as Reynolds' own interpretation would have it. Such an interpretation seems most likely in windy conditions when the LCS take the form of long, streamwise roll vortices whose heights span the CBL and, being semi-permanent, provide a kind of physical embodiment of the mean wind.

The key consideration here is that the LCS are semi-permanent, which is to say steady structures on time and length scales long compared to all turbulence time and length scales within the CBL (Brown, 1980). They may be regarded as a physical embodiment of the mean wind, ${}^t\overline{u}(y,z)$, in any single realization of the flow, where 't—' indicates time averaging over an interval long compared to all turbulent time scales. The y coordinate appears along with z because there will be lateral as well as height variations in the local velocities within the roll structures. Now we can, from first principles and for a single realization of the flow, associate $-{}^t\overline{u'w'}\ \partial {}^t\overline{u}(y,z)/\partial z$ with the divergence in the downwards flow of kinetic energy from the LCS to the shear-produced eddies of the SFL, so we can associate it with a particular pathway in Fig. 1. This is similar to, but not identical to, the term $-{}^{\Xi}\overline{u'w'}\ \partial^{\Xi}\overline{u}(z)/\partial z$ that appears in the RANS energy equation, where we now use 'Ξ—' to indicate ensemble averaging explicitly. Indeed the terms are very similar if the ensemble is constructed from flow realizations in which the positions of the LCS are identical in each realization, but we do not obtain exactly the term found in the RANS energy equation since it applies at only one lateral position. That is, the ensemble-averaged term retains the y-dependence of the time-averaged term. To address this we must consider an ensemble constructed so that the roll structures have random lateral positions. This is the kind of ensemble envisaged in the RANS equations.

Before examining this case we first write some definitions and relationships between ensemble and time averages. We write two particular equations for the differences in these

$$ {}^{t}\overline{u'w'} = {}^{\Xi}\overline{u'w'} + \Delta\tau, \tag{12} $$

and

$$ {}^{t}\overline{u} = {}^{\Xi}\overline{u} + \Delta\overline{u}, \tag{13} $$

where now

$$ {}^{\Xi}\overline{\left({}^{t}\overline{u'w'}\right)} = {}^{\Xi}\ \overline{u'w'}, \tag{14} $$

and

$$ {}^{\Xi}\overline{(\Delta\overline{u})} = {}^{\Xi}\ \overline{u}, \tag{15} $$

and in consequence

$$ {}^{\Xi}\overline{\Delta\tau} = 0, \tag{16} $$

and

$$ {}^{\Xi}\overline{\Delta\overline{u}} = 0. \tag{17} $$

Here the Δ terms indicate the difference between the corresponding time-averaged and ensemble-averaged terms, which depend on y. With these we can explore the ensemble average of our time-averaged expression ${}^{t}\overline{u'w'}\ \partial^{t}\overline{u}(y,z)/\partial z$, and so write

$$ {}^{\Xi}\overline{\left({}^{t}\overline{u'w'}\ \frac{\partial^{t}\overline{u}(y,z)}{\partial z}\right)} = {}^{\Xi}\overline{u'w'}\ \frac{\partial^{\Xi}\overline{u}(z)}{\partial z} + {}^{\Xi}\overline{\left(\Delta\tau(y,z)\ \frac{\partial\Delta\overline{u}(y,z)}{\partial z}\right)} \tag{18} $$

The second term on the right-hand side is the lateral covariance between the wind shear and the shear stress. It may not be small since the correlation between $\tau(y,z)$ and $\partial\Delta\overline{u}(y,z)/\partial z$ will be very high. To the extent that it is significant it prevents a simple identification of an ensemble-averaged, horizontally-uniform wind speed, as usually defined, with the streamwise velocity of the roll vortices. In strong wind conditions $-{}^{t}\overline{u'w'}\ \partial^{t}\overline{u}(y,z)/\partial z$ can be associated with the divergence in the energy shown flowing down from the LCS into the SFL in Fig. 1. The same identification cannot be made for ensemble averages.

In light wind conditions, when the LCS take the form of polygonal convection cells rather than long streamwise roll vortices, it becomes impossible to make any simple identification of the mean wind with the velocity field within the LCS, and so impossible to interpret $-\overline{u'w'}\ \partial\overline{u}(z)/\partial z$ formally as the effect of the mean motions of the LCS on the turbulence within the SFL. The convection cells remain semi-permanent but they move with the wind in the CBL. An Eulerian observer, on a tower or in a tethered balloon, would experience them as erratic wind fluctuations indistinguishable from the largest turbulent structures, and would observe their energy spectrum to peak at z_i-scale, in the same region as the largest turbulent eddies formed from instabilities in the roll vortices of windier conditions. Indeed, one might interpret the emergence of the cellular circulations as the largest turbulent eddies becoming more organized as they take over the role of principal heat transporters in the CBL as wind speed decreases. As this happens we loose our ability to separate the LCS and the largest turbulent eddies, and so lose our ability to associate a local mean with motions of the LCS. Without a clear definition of $\partial\overline{u}/\partial z$ at each point and moment in a single flow realization it is not possible to identify $-\overline{u'w'}\ \partial\overline{u}(z)/\partial z$ uniquely with any distinct energy flow pathway in Fig. 1.

In summary, despite further analysis we have made very little progress in developing separate interpretations for the terms $-\overline{u'w'}\ \partial\overline{u}/\partial z$ and $-\partial\overline{w'e}/\partial z$ of the RANS energy equation, Eq. 7, within the SFL. Before commenting on the significance of this we examine the situation above the SFL since the energy flows are different there.

4.2 Interpretation Above the SFL

SFM interprets the first term of Eq. 7 as *shear production of TKE* regardless of height in the CBL. It is, of course, true that there are downwards momentum fluxes and positive velocity gradients at both the top and the bottom of the CBL, so $-\overline{u'w'}\ \partial\overline{u}(z)/\partial z$ is large and positive at both levels, but this does not mean that the underlying physical processes are the same. Indeed some energy flows are different within and above the SFL. For example, the work done against surface drag within the SFL has no counterpart near the top of the CBL (Fig. 1), and the entrainment energy flux at the top of the CBL has no counterpart at its bottom, within the SFL. Again our question is whether, in this new location, we can associate the terms $-\overline{u'w'}\ \partial\overline{u}/\partial z$ and $-\partial\overline{w'e}/\partial z$ with distinct energy flow pathways in Fig. 1.

Before continuing we should consider the semantic question of whether entrainment should be regarded as a turbulence process at all, even though it is certainly a feature of the turbulent CBL system. Entrainment is the process whereby air from the free atmosphere above the CBL is drawn down, by a pressure gradient, into the common-down areas between the LCS, as hinted in the diagram of Wyngaard (1985), or into the centres of convection cells in windless conditions. In windy conditions, when the LCS are helical roll vortices, the entrained air is introduced into the CBL as long, streamwise sheets of unmodified air, though these are quickly broken up and dispersed by the turbulence processes acting within the CBL, and so quickly incorporated into the CBL. The motions of the helical roll vortices is steady over all turbulence time scales, and they contribute nothing to turbulence energy spectra. That is why Fig. 1 shows these as 'coherent structures', which they certainly are by any definition, while the other boxes of the diagram contain 'eddies' in which unsteadiness (chaotic motion) but not coherence is a necessary characteristic. These 'largest turbulent eddies give rise to a continuous spectrum of turbulent fluctuations in the production region of the outer energy spectra at z_i scale. Thus entrainment involves the introduction of non-turbulent air from above the CBL into the CBL by a steady pressure mechanism associated with the rotation of the roll vortices. In this sense entrainment is not a turbulent process even though the entrained air, even as it is being introduced, is dispersed by turbulence processes acting within the CBL. These smaller-scale processes mediate the transfer of the kinetic energy and momentum entrained from above into the LCS. Whatever words are used to describe it, the concept of *shear production of TKE* has no relevance to the entrainment process.

As an aside (because Fig.1 assumes windier conditions) we note that somewhat similar considerations apply to the cellular convection cells that appear in windless conditions. Convection cells are also stationary structures, being long-lived on the scale of their own turnover times. Once again, entrained air is drawn downwards by a vertical pressure gradient, now into the central regions of the polygons. In light winds convection cells move with the mean wind in the CBL, and may be only weakly aligned into rows along the wind. This transitional case of light winds blurs the distinction drawn between the LCS and the largest turbulent eddies in Fig. 1 and will not be discussed further here.

The wind speed above the CBL is greater than that within it, so entrainment transfers both momentum and kinetic energy down into the CBL. The momentum goes, initially, to the long helical roll vortices, to maintain their speed against losses to surface drag, while pressure redistributes the momentum and energy within them to maintain the form of the convective rolls (Zhuang, 1995). The entrained air is slowed progressively as it is drawn down and mixed into the CBL, so entrainment is associated with a positive velocity gradient over a significant height range in the upper part of the CBL. Part of the velocity gradient we see in experimental results, and in results from large-eddy simulation (LES) studies, is also due to fixed-level averaging of processes near a lumpy CBL top, so we are uncertain about the depth of this layer. The downwards momentum flux and positive velocity gradient make the first term in Eq. 7 positive, but the process can hardly be described as *shear production of TKE* since the shear is the result, not the immediate cause of entrainment, and no TKE is 'produced'. Because the form and forward motion of the LCS must be maintained equally in all parts of those structures, pressure must redistribute the entrained mechanical energy evenly with height, so we expect pressure transport to be downwards and uniform over the middle part of the CBL. This is in agreement with the LES results of Pino et al. (2003) (their Fig. 7).

The entrained air, drawn down into the CBL from above the capping inversion, is warmer than that within the CBL, so its introduction is opposed by buoyancy forces.

Some of this opposition may appear in the term $\partial\overline{w'p'}/\partial z$ in the RANS energy equation, Eq. 7, (just as it does within the SFL, albeit with a reversed sign), and some may appear in the advection terms since vertical advection of kinetic energy is not excluded at the undulating upper boundary of the CBL. The LES results of Pino et al. (2003) (their Fig. 7) show a small peak in downwards pressure transport at the top of the CBL, in support of this interpretation. We would not expect the rate of dissipation to be elevated at the top of a CBL since the entrainment process involves only large, non-turbulent structures not directly connected to any Richardson cascade. The LES results of Pino et al. (2003) show little excess dissipation in the upper CBL, neither do the experimental results of Kaimal et al. (1976) (though it is not clear how many of their measurements were made within the entrainment layer). The LES results of Conzemius and Fedorovich (2006) do show elevated dissipation in the entrainment layer, so some clarification is needed.

Once again, Fig. 1 provides a qualitative interpretation of CBL processes that is in good agreement with LES and experimental results throughout the CBL. In the entrainment layer we cannot associate $-\overline{u'w'}\ \partial\overline{u}(z)/\partial z$ with *shear production of TKE* since the entrainment process is not driven directly by the shear, and the entrained energy goes directly to the LCS, which are, arguably, not turbulent structures. In the entrainment layer, once again, the standard interpretation of terms offered by SFM is not a reliable guide to the turbulence processes.

5 Towards Modelling the CBL

Modelling is about predictability, but predictability in CDS is a very different thing to that understood by the physicists of the nineteenth century (Kellert, 1993). Then it was believed that the physics of a problem was understood once a mechanical model of it was constructed, after which particular solutions could always be produced by solving the governing equations subject to certain initial and boundary conditions. In this sense the problem of turbulence should have been solved once the Navier-Stokes equation had been written. However, the exponential growth of errors in chaotic systems, such as that defined by the Navier-Stokes equations and found in the CBL, precludes this kind of predictability. Any integration of the governing equations (i.e. any particular flow simulation) will, as time passes, give detailed results that are less and less like that of a target flow, even when the initial states are matched as closely as possible. The best that can be hoped is that the simulated flow will have a very similar strange attractor to the target flow, and so very similar statistical properties to the target flow. If so, research could proceed empirically by exploring how the statistical properties of simulated flow respond to variations in control parameters. For example, we could investigate how CBL growth responds to changes in the strength of the capping inversion, the surface heat flux, the surface roughness length. In its simplest form, this kind of investigation would be wholly empirical and descriptive—without benefiting in any way from our knowledge of the underlying dynamics. The question is, can we do better than this? Can we model the CBL in a way that makes use of at least some of our understanding of how real CBL flows work? Can our higher-level understanding of eddy processes in the CBL help us in modelling?

The empirical evidence is that it can, and for this we turn to the defining property of CDS: their ability to generate regular and statistically-predictable patterns of motion. Coherent structures and eddies represent a meeting point of the individual and ensemble-averaged conceptions of a flow since eddies can be understood dynamically, at least when they are isolated and idealized, while their ensemble properties can be described empirically using data from real flows or simulations. The key properties that allow this are that each class of eddies has, under a range of conditions, rather stable and reproducible characteristics, so that eddies can be identified; and different kinds of eddies of similar scale cannot exist in the same place at the same time, because non-linear systems do not allow the linear superposition of eddies of similar scale, so they (or their ensemble properties) can be isolated. These properties are sufficient to allow turbulent spectra and cospectra, for example, to be divided into subranges in which each subrange has particular characteristics that reflect the particular properties of each particular class of eddies. This means that velocity and scalar spectra can display different scaling behaviour in different wavenumber ranges, as is well known in fluid mechanics. Identification of the scaling parameters then tells us something of the origins and nature of each particular kind of contributing eddy. This provides us with a powerful tool for studying eddies, and ultimately for modelling their roles and properties in the turbulent system.

Eddies have three key properties: form, size and mechanical energy density. The first of these cannot be predicted by any known method, which means that any practical model must be at least partly empirical. However, given a form we can use a length scale, $\mathcal{L}$, to represent size, and given a length scale we can write an energy scale as $(\mathcal{L}\mathcal{E})^{2/3}$, where $\mathcal{E}$ is the flow of mechanical energy maintaining that particular population of eddies. These are the elements needed to develop eddy similarity models: models that are similar in ambition to the Monin-Obukhov similarity model but have ensembles of like eddies as their subject, rather than whole flows. The hope is that the universal constants and functions of the models will, this time, prove to be truly universal. Recent experimental results from the CBL give cause for optimism (McNaughton et al. 2007; Laubach and McNaughton, 2009). These parameterized eddies can then comprise the parts of an 'eddy-mechanics' models of the CBL system.

It is too early to lay out a detailed plan for building CDS-based models of the CBL, but some of the elements can be foreseen. We will have to solve the momentum, energy and heat flows in the CBL simultaneously. Solving for the energy flows in the CBL is particularly important, since the mechanical energy flows dictate the energies of the various classes of eddies. To solve this problem we will need to model both entrainment from above the CBL, and transport within the CBL. This will require sub-models based on the actions and connections of the various coherent structures and eddies in the CBL. Our knowledge of these eddies and their action will be semi-empirical, in the form of eddy-similarity constants and functions. An important internal parameter of the solution for the CBL will be the height of the SFL, z_s. Knowledge of transport within this layer is particularly important since the largest gradients of wind speed and temperature are found within the SFL. Processes there will have to be modelled in terms of the three classes of eddies active in the SFL: small detached eddies of the inner Richardson cascade, which apparently act diffusively (Wyngaard and Coté, 1972); upscale cascade structures in the SFL, named TEAL structures by McNaughton (2004), which transport momentum and whose action is driven by both the mean and fluctuating components of shear; and impinging outer eddies, which do not depend on shear but organize the locations and strengths of the TEAL structures and, as height increases towards the top of the SFL, limit the numbers of TEAL structures and take over their transporting role (Laubach and McNaughton, 2009). It must be expected that there will be no unique solution to this problem unless an optimization principle is found, for which the maximum entropy production theorem is the likely candidate, though this has yet to be formulated in a manner applicable to CBL studies.

The role of the RANS equations will likely be limited in this new kind of modelling because they give a height-specific description of processes without discriminating the kinds of eddies active at each height, while the underlying physics attaches to processes that are eddy-specific and may be distributed in height.

Lest this might seem an unduly daunting, or even an unnecessary task, let me quote from a review of turbulence modelling by Lumley and Yaglom (2001)—two of the leading proponents of the SFM paradigm in the second half of the twentieth century. In their millennial review of progress they wrote: "*even after 100 years, turbulence studies are still in their infancy. We are naturalists, observing butterflies in the wild. We are still discovering how turbulence behaves, in many respects. We do have a crude, practical, working understanding of many turbulence phenomena but certainly nothing approaching a comprehensive theory, and nothing that will provide predictions of an accuracy demanded by designers.*" This is close to an admission of failure of the SFM enterprise, at least as a guide for further progress. CDS offers a new way forward. Our particular 'butterflies' are the patterns of motion (eddies) that characterize turbulence, and the task ahead is to learn more about them, and to use this knowledge to model important flows such as the CBL.

6 Conclusions

This paper has discussed turbulence in the convective atmospheric boundary layer at a rather fundamental level, in the hope that this will throw some light on the state of our science. We have discussed the production and flow of turbulence energy in the CBL using two quite different paradigms. The first, the statistical fluid mechanics (SFM), includes the interpretations developed by Reynolds (1895), and adopted by the mainstream of boundary layer meteorology ever since. SFM provides a means of understanding turbulent flows because it associates the causal relationships embodied in the Navier-Stokes equations with the local interpretation of the individual terms of the various RANS energy equation. Thus, in SFM, 'shear production of TKE' is interpreted as a real, local process—Reynolds' *cause of transformation*—which transfers kinetic energy from the mean flow to the fluctuations by an unspecified physical mechanism. The RANS equations have been widely used to model boundary-layer flows, and they underlie the many studies of local budgets of turbulence energy in convective boundary layers. Of particular significance in these studies is the buoyant production term of the RANS energy equation which has wrongly been interpreted by Richardson (1920) and Monin and Obukhov (1954) as the local transfer of energy to the motions of the eddies that are responsible for transport of both heat and momentum. SFM leads to internal contradiction since the term $-\overline{w'u'}\ (\partial\overline{u}/\partial z)$, which is unambiguously the largest part of the divergence in a downwards flux of kinetic energy near the ground, is interpreted everywhere as a transfer of kinetic energy from the mean flow to the eddies at the same level. We have traced this problem back to the failure of Reynolds' original arguments for the physical interpretation of the RANS energy equation.

The second paradigm, the complex dynamical systems (CDS), collects together several lines of work, some of which have not previously been identified as such in boundary-layer studies. Complex dynamical systems are pattern-forming systems, and CDS models take a fundamentally deterministic, albeit chaotic view of turbulence. CDS has a shorter history and has yet to contribute significantly to the development of practical models. The CDS paradigm leads to an emphasis on energy flows from one class of flow structure to another, which flows have been neglected in boundary-layer meteorology. The paper presents a schematic diagram representing the flows of mechanical energy in the convective boundary layer. These energy flows are important because they can be used, with a characteristic eddy length scale, to parameterize the energy of eddies, and so provide a foundation for similarity modelling of eddy transport processes. The CDS paradigm denies the possibility of any strictly local interpretation of transport processes in turbulent flows, and so is incompatible with the SFM interpretation. The CDS approach is used to provide a critique of the usual SFM-based interpretation of the RANS energy equation.

We conclude that statistical fluid mechanics in the Reynolds interpretation is fundamentally flawed as a means of understanding physical processes in turbulent flows, such as the convective boundary layer. Complex dynamical systems, on the other hand, offers a more reliable framework for understanding boundary-layer flows, and provides a possible new way forward for constructing better practical models of the CBL.

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