# UNSTEADINESS AS A CAUSE OF NON-EQUALITY OF EDDY DIFFUSIVITIES FOR HEAT AND VAPOUR AT THE BASE OF AN ADVECTIVE INVERSION

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**Abstract.** This paper examines the effect of non-stationarity of the wind on similarity of the eddy diffusivities for heat and vapour within a stable layer at the bottom of an internal boundary layer formed downwind of a dry-to-wet transition. First, we present some experimental data taken above a rice crop downwind of very extensive dry range lands at Warrawidgee, NSW, Australia. These data establish that periods of higher wind speed were associated with periods of higher saturation deficit in the canopy of the rice crop, and lower Bowen ratio. It is shown that Bowen ratios calculated for 30-second sub-intervals varied three-fold within a single 20-minute averaging period. Thus periods of higher wind speed corresponded to periods of higher moisture flux and smaller sensible heat flux.

An idealized situation is then analysed theoretically. It is assumed that the time scale of the slow variations of the wind is long compared with the surface-layer time scale and that fetch is sufficient that the air near the ground is in continuous equilibrium with the surface. Using a two-scale Reynolds decomposition of the fluctuating wind and scalar variables into active and inactive components, it is shown that unsteadiness can lead to an eddy diffusivity for saturation deficit, calculated as the ratio of average flux to average gradient, that is larger than that for total energy calculated in a similar way. Using this ratio to calculate the ratio of diffusivities for temperature and humidity, $K_T/K_q$, it is found that the latter can be much larger than one if the Bowen ratio is small and negative. Despite this, assuming $K_T = K_q$ and using the Bowen ratio method to calculate surface energy fluxes will usually incur only minor errors.

**Keywords:** Advection, Advective inversion, Eddy diffusivity, Internal boundary layer, Monin-Obukhov similarity, Unsteadiness

## 1. Introduction

Non-stationarity lurks in much of what we do in surface-layer meteorology, but it is rarely acknowledged. In applied micrometeorological experiments particularly, where true stationarity is an unexpected luxury, clouds come and go while the wind veers, backs and changes strength on time scales long compared with the surface-layer turbulent time scale, $z/u_*$, but comparable to typical averaging intervals for measurements. Despite this, Monin-Obukhov similarity theory, or methods based on it, remains the paradigm for data analysis and interpretation even while stationarity – a basic assumption of the theory – is routinely violated. It is a moot point whether or not this matters because scarcely any work has been done on the consequences of non-stationarity on relationships between time-averaged

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quantities in the surface layer. (For a rare example see Stearns, 1971). The present paper addresses such a question: whether the eddy diffusivities for temperature and humidity, $K_T$ and $K_q$, which are calculated from time-averaged temperature and humidity fluxes and gradients, are affected by slow variations in the driving winds. We consider the ratio, $K_T/K_q$, within an internal adjusted layer (IAL) formed at the base of an advective inversion downwind of a sharp dry-to-wet transition. The possible inequality of the eddy diffusivities for heat and vapour has been a contentious issue for this situation. We begin our study of it with a brief review of conditions which may cause inequality of scalar diffusivities, moving on to a discussion of non-stationarity in later sections.

The eddy diffusivities for all conserved scalars are usually assumed equal in the turbulent surface layer of the atmosphere. The argument given for this is that they all are carried by the same eddies because they are associated at the source (Swinbank and Dyer, 1967). This is a powerful argument. If two conserved scalars are associated at the source ('loaded onto the same eddies') then they must remain perfectly correlated throughout the flow since no purely turbulent mechanism can disassociate them thereafter. If two scalars are perfectly correlated at a point in a flow, so that $p = a + bq$, then multiplying by $w'$ and averaging shows their eddy fluxes must always be related by $\overline{w'p'} = b\overline{w'q'}$, and the instantaneous local gradients must always be related by $\partial p/\partial x_i = b\partial q/\partial x_i$ or else small turbulent displacements of air would produce fluctuations which break the perfect correlation. Thus the mean gradients must be related by $\partial \bar{p}/\partial x_i = b\partial \bar{q}/\partial x_i$ from which we deduce that their diffusivities are equal: $K_p = K_q$. This similarity obtains whether the turbulent flow is simple, as within an equilibrium surface layer above an extensive level plane where the rules of Monin-Obukhov similarity apply (Dias and Brutsaert, 1996), or complex, as over topography or through barriers; it applies whether or not the flow is influenced by pressure gradients or buoyancy; it also applies when the fluxes are variable in space or time (non-stationary) provided the scalar substances issue from their sources in fixed ratio, $F_p/F_q = b$. Perfect association of scalars at source implies perfect correlation of fluctuations which, in turn, implies equal eddy diffusivities.

Eddy diffusivities need not be equal where sources are differently distributed so that scalar fluctuations are imperfectly correlated. For example, in the 'mixed layer' which lies above the surface layer in a convective boundary layer (CBL), the rising air of the updrafts carries positively-correlated temperature ($T$) and specific humidity ($q$) fluctuations originating in the surface layer below, while the subsiding air carries negatively-correlated $T$ and $q$ fluctuations originating in the entrainment layer at the top of the CBL. A stationary observer at a mid level in the CBL sees fluctuations from both sources and records small overall $T$-$q$ correlation, $|R_{Tq}| \ll 1$ (Wyngaard et al., 1978). In such conditions the net sensible heat flux is upwards along a near-neutral or even slightly inverted mean gradient of potential temperature while the vapour flux generally moves down small lapse humidity gradients. Eddy diffusivities for $T$ and $q$ are not equal (Wyngaard and Brost, 1984).

Unequal eddy diffusivities can also be found in the vegetation sublayer at the base of the surface layer. For example, Denmead and Bradley (1985) found that eddy diffusivities for heat and water vapour frequently had opposite signs within the sub-canopy space of a 16-meter tall pine forest. This non-similarity was associated with small correlation coefficients between the fluctuations in $T$, $q$ and $CO_2$ concentration, $c$ (Denmead and Bradley, 1987; pers. com. O.T. Denmead, 1997). The sizes of the correlation coefficients $|R_{Tq}|$ and $|R_{cq}|$ measured on selected days just below canopy top at 15 m and in the trunk space at 3 m were mostly less than 0.5; $R_{cq}$ was usually negative near the top of the canopy but positive near the ground. These small correlation coefficients reflect the differently distributed sources of the three scalars, particularly differences in their ratios in the overstorey and at the ground. At 10 m above the canopy top $|R_{Tq}|$ was still much less than one, being mostly in the range 0.6–0.9 during the middle of the day. Even so, the temperature and humidity profiles had the same shape down through that level (Denmead and Bradley, 1985) and the diffusivities were equal there. Imperfect correlation between scalars is, therefore, a necessary condition for dissimilarity in flux-gradient relationships, but not a sufficient one.

The above two examples of dissimilarity are from layers of the atmosphere above and below the surface layer, in situations where the scalar fluxes are emitted in different ratios at the top and bottom of the layer. A third example concerns the surface layer itself, in advective situations where the ratio of scalar fluxes from the surface varies in the streamwise direction. The simplest and most studied case of scalar advection deals with wind blowing from one uniform area across a sharp boundary and over another area where heat and vapour fluxes are emitted in different ratio (e.g. Rider et al, 1963; Lang et al, 1983; De Bruin et al., 1991). An observer over the downwind area finds imperfect $T$-$q$ correlation and $K_T \neq K_q$ (e.g. Bink, 1996a). This is easily explained if our observer is located in the outer part of the internal boundary layer (IBL) developed over the downwind area. We can argue that poor correlation arises because he sees advected $T$ and $q$ fluctuations which are in the ratio of the upwind Bowen ratio superimposed on locally-produced fluctuations which are in the ratio of the downwind Bowen ratio. Our observer sees different diffusivities (Bink, 1996a,b) because he sees advected $T$ and $q$ fluxes and gradients superimposed on downwind-generated fluxes and gradients, the latter having a smaller diffusivity because the length scale of the transporting eddies is smaller for the fluxes generated downwind; the ratio of the diffusivities calculated from the total fluxes and gradients will take a value which depends on both the ratio of the two separate diffusivities and the ratios of the upwind- and downwind-generated gradients, as shown in Appendix A.

This argument for non-equality of eddy diffusivities applies to the outer part of the IBL where scalar fluxes and gradients are known to be influenced by both up- and down-wind sources. Near the ground, so the argument goes, the fluxes and gradients advected from the upwind source area must both approach zero; the locally-produced fluxes and gradients then predominate and eddy diffusivities are again equal. That is, there forms an internal adjusted layer which is in dynamical equilibrium with the new surface and where the micrometeorological elements again obey Monin-Obukhov similarity (Garratt, 1990). Such an assumption is widely used in numerical modelling to construct lower boundary conditions (e.g. Rao et al., 1974).

There are, however, a number of papers which report marked non-similarity of scalar transport near the bottom of an IBL, at levels where we might expect fluxes and gradients to obey Monin-Obukhov similarity; this must imply small $|R_{Tq}|$ values. These results come from three experimental sites: Mead in USA (Blad and Rosenberg, 1974; Verma et al., 1978; Motha et al., 1979), Griffith in Australia (Lang et al., 1983) and La Crau in France (de Bruin et al., 1991; Bink, 1996a). The workers at Mead all report $K_T > K_q$ over extensive areas of alfalfa or soybeans in daytime conditions where 'regional' inversions were common (i.e. the temperature profile was in inversion up to at least 16 meters). In contrast to this, workers at the other sites report $K_T < K_q$ within an advective inversion downwind of sharp transitions from arid land to irrigated crops of rice (Griffith) or grass (La Crau). The explanation given by Lang et al. (1983) is that, even at this low level, larger-scale eddies brought down air from levels where temperature and vapour profiles were distinctly different, so the transport effectiveness of the larger eddies is quite different for heat and vapour. In effect, they argue that the fetch-to-height ratio in this experiment, though large, was insufficient to establish Monin-Obukhov similarity at instrument level. This conclusion is supported by probability analysis of the transport processes at La Crau by Kroon and Bink (1996), and by the calculations of Bink (1996a) using the second-order-closure model of Rao et al. (1974). This mechanism will always lead to $K_T < K_q$ within an advective inversion downwind of a dry-to-wet transition (Appendix A).

Curiously, this mechanism does not explain the results from Mead. At Mead $K_T$ was found greater than $K_q$, not less, in inversion conditions and the effective fetch (including similar fields surrounding the experimental site), though ill-defined, seems to have been very much greater than at Griffith or La Crau judging by the depth of the advective inversion. Experimental error at Mead, though always possible, is not a satisfying explanation because similar results were obtained in three different seasons and using two different systems for flux measurement: lysimeter and energy balance for the first two experiments and eddy correlation instruments for the third. There is, it seems, more to be learnt about transport similarity within an advective inversion at very large fetch.

A good place to start an investigation is to ask how $|R_{Tq}|$ might become less than one in the air layer close above a surface that is uniform for some considerable distance upwind. One suggestion was made by De Bruin et al (1993), who proposed that observed scalar fluctuations near the ground include a component generated by larger-scale processes. De Bruin et al. (1993) found that wind and temperature fluctuations measured at 10 m over the semi-arid plane at La Crau followed the usual Monin-Obukhov-similarity dependence on height and stability, but that the humidity fluctuations did not. They also found small $|R_{Tq}|$ values. Humidity fluxes and gradients were both small near the ground so locally produced humidity variance was also rather small. They proposed that the humidity variance carried down from the entrainment layer at the top of the convective boundary layer, though small in itself, then comprised a significant fraction of the total humidity variance near the ground. For temperature the locally-produced variance was large enough to overwhelm that carried down from above. Essentially, their argument is that quantities such as $\sigma_T/T_*$ and $\sigma_q/q_*$, like $\sigma_u/u_*$, do not strictly obey Monin-Obukhov similarity at the base of a convective boundary layer, but that the departures are not easily noticed unless $T_*$ or $q_*$ is quite small.

We can visualize this situation by imagining that we mark the scalar entity released at the surface, $p_a$, with a red tracer and mark the externally-generated scalar, $p_p$, with a yellow tracer. Since there is a flux of red scalar at the ground, its fluctuations are correlated with the vertical wind, $w$, so that $\overline{w'p'_a} = F_{p,a}$ and we describe these fluctuations as active (flux-carrying). The yellow fluctuations, on the other hand, carry no flux to the ground, so the yellow fluctuations are uncorrelated with $w$ near the ground and $\overline{w'p'_p} = 0$; we describe the yellow fluctuations as inactive. Close above a uniform surface the red tracer conforms to the norms of Monin-Obukhov similarity theory so, for example, the length scale of the red fluctuations will scale with height. The yellow tracer, on the other hand, will not observe Monin-Obukhov similarity near the ground and the size of yellow-traced eddies will not scale with height. The power spectra of the red and yellow fluctuations must become increasingly different as the ground is approached. With sources widely separated, a reasonable expectation is that active and inactive fluctuations become quite uncorrelated near the ground, so $\overline{p'_a p'_p} = 0$. Also, flux-profile relationships for red tracer will behave according to the precepts of Monin-Obukhov similarity, unaffected by the yellow tracer. Extending the argument, we could have two perfectly-correlated inactive scalars from upwind, say $T_p$ and $q_p$, superimposed locally on another two active scalars, $T_a$ and $q_a$ which are also perfectly correlated, but whose linear relationship has a different slope. In such a case we would expect the observed local correlation $|R_{Tq}|$, i.e. the correlation between $(T_a + T_p)$ and $(q_a + q_p)$, to be less than one even though $T_a$ and $q_a$ obey Monin-Obukhov similarity in all respects, including having equal eddy diffusivities. Since in the internal adjusted layer the inactive parts of the fluctuations of $T$ and $q$ contribute to neither flux nor gradient of the total $T$ and $q$, by definition, equality of eddy diffusivities for $T_a$ and $q_a$ must extend to equality of the diffusivities of total $T$ and total $q$.

We draw two conclusions from this review: firstly, while a small correlation coefficient $|R_{Tq}|$ is a necessary condition for non-similarity ($K_T \neq K_q$), it is not a sufficient one; secondly, the superposition of inactive fluctuations onto the local, active scalar is not, by itself, enough to induce non-similarity within the IAL. If, as claimed, $K_T$ is not equal to $K_q$ close above a uniform, well-watered surface well downwind of a dry-to-wet transition then there must be some other way to disrupt the perfectly associated issuance of heat and vapour from their common source at the ground. A possibility for this is that heat and vapour sources are not perfectly associated in time, even within a normal averaging period of, say, 20 minutes, for which stationarity is normally assumed. This possibility is the main subject of our paper; we examine it below.

## 2. Some Interesting Data

Temporal variation of surface fluxes has rarely been studied so we begin with some data to indicate that this is an interesting subject and worthy of closer examination. Figure 1 shows a scatter plot of temperature vs. humidity measured at 10 Hz during a 20-minute period over a field of paddy rice at Warrawidgee. Upwind lay the extensive dry range lands of Wyvern Station and beyond that further dry scrub and grasslands stretching for hundreds of kilometres. Full details of site and instrumentation for this experiment will be reported elsewhere. Suffice to say that the measurements were made in summer under light winds and steady net radiation with a fetch-to-height ratio of about 250:1.

Figure 1 shows rather poor overall correlation between the $T$ and $q$ fluctuations, with $R_{Tq} = -0.76$. Closer examination of the data reveals something more interesting. Regression analyses performed on data from successive 30-second subintervals typically showed much higher correlation, but the slopes of the short-period $T$ vs $q$ relationships varied considerably over the whole period. Because of the high correlation within the short intervals, $c_p/\lambda$ times the slopes of the fitted regression lines during those intervals can be interpreted as brief-period Bowen ratios. They are illustrated in Figure 2, along with smoothed time series for some other meteorological variables.

Figure 2 shows that there is a slowly-varying component to $T$ and $q$, providing evidence that inactive fluctuations of $T$ and $q$ were superimposed on the active fluctuations during this experiment. These slow fluctuations were evident right down within the rice canopy, where they could directly influence the transpiration rate. The data also show that the magnitude of the short-period correlation coefficients, $|R_{Tq}|$, was usually quite near one, as would be expected in an equilibrium layer, but that the slope of the $T$ vs $q$ relationship varied considerably over the longer period. These observations lend support to an hypothesis that there exist active fluctuations of $T$ and $q$ which are very well correlated from one brief period to the next, but that the ratio of the 30-s-averaged heat and vapour fluxes varies continuously in time. This is consistent with the idea that active transport of $T$ and $q$ is similar over short periods, in conformity with Monin-Obukhov similarity. It also suggests that there may well be a problem in applying Monin-Obukhov similarity to fluxes and gradients averaged over longer periods since the fluxes are not steady.

We can now make an informed guess as to the source of the slow fluctuations. During our experiment we observed very small humidity fluctuations when the

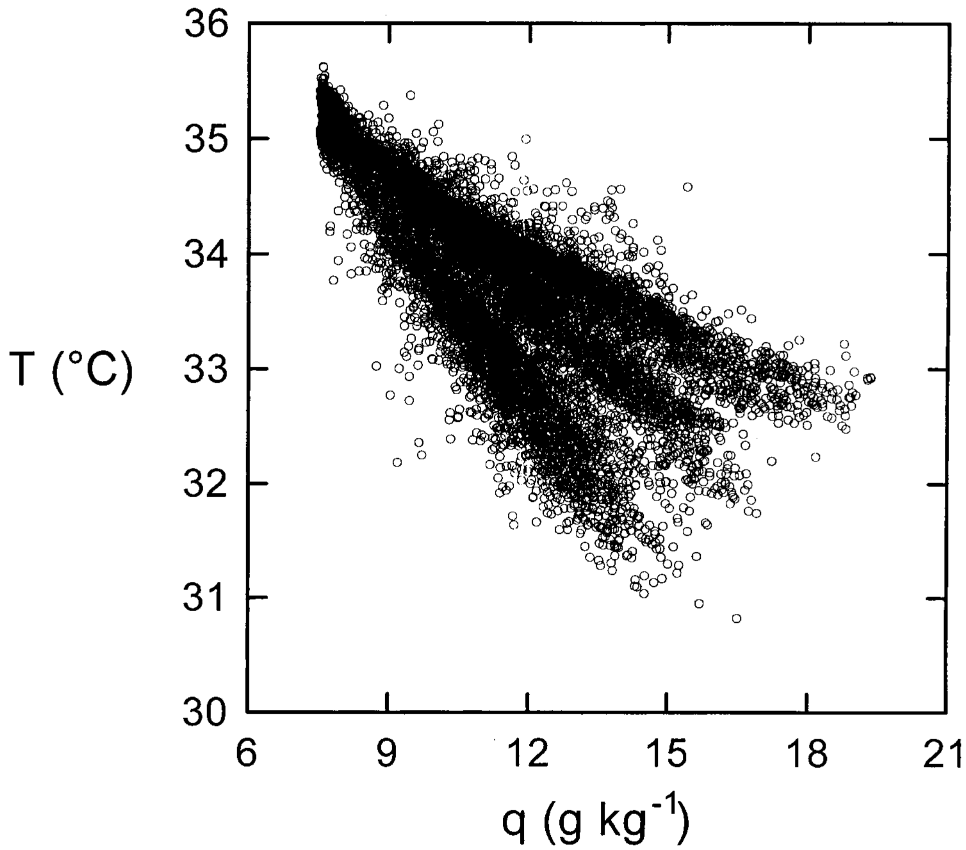


*Figure 1.* Scatter plot of observations of temperature, $T$, and humidity, $q$, taken over paddy rice in the Warrawidgee district of the Murrumbidgee Irrigation Area, 30 km west of Griffith in New South Wales, Australia on 5 February, 1997. Data were collected at 10 Hz during a 20-minute period starting at 1441 Eastern Daylight Time when net radiation was nearly constant. The instruments were sited at 2.4 m above water level (about 2 m above the displacement plane of the crop) with fetch of about 460 m. The correlation coefficient, $R_{Tq}$, is $-0.76$ for these data.

wind blew across bare fields to the east – our instruments being very close to the eastern edge of the field. We expect similarly small fluctuations in the incident humidity at the western edge on westerly winds. Therefore the humidity fluctuations we observed on west winds were not simply created upwind and advected across the site as superimposed inactive fluctuations, but must have been produced over the rice. Also, Figure 2 shows that the slow variations in saturation deficit within the rice canopy were well correlated with the filtered variations in wind speed, which we must suppose had their origin in CBL-scale convective activity over the dry upwind plane. We deduce that changes in forcing by the larger-scale winds caused the slow fluctuations in $T$ and $q$ by changing development of the horizontal and vertical $T$ and $q$ profiles within the IBL over the paddy.

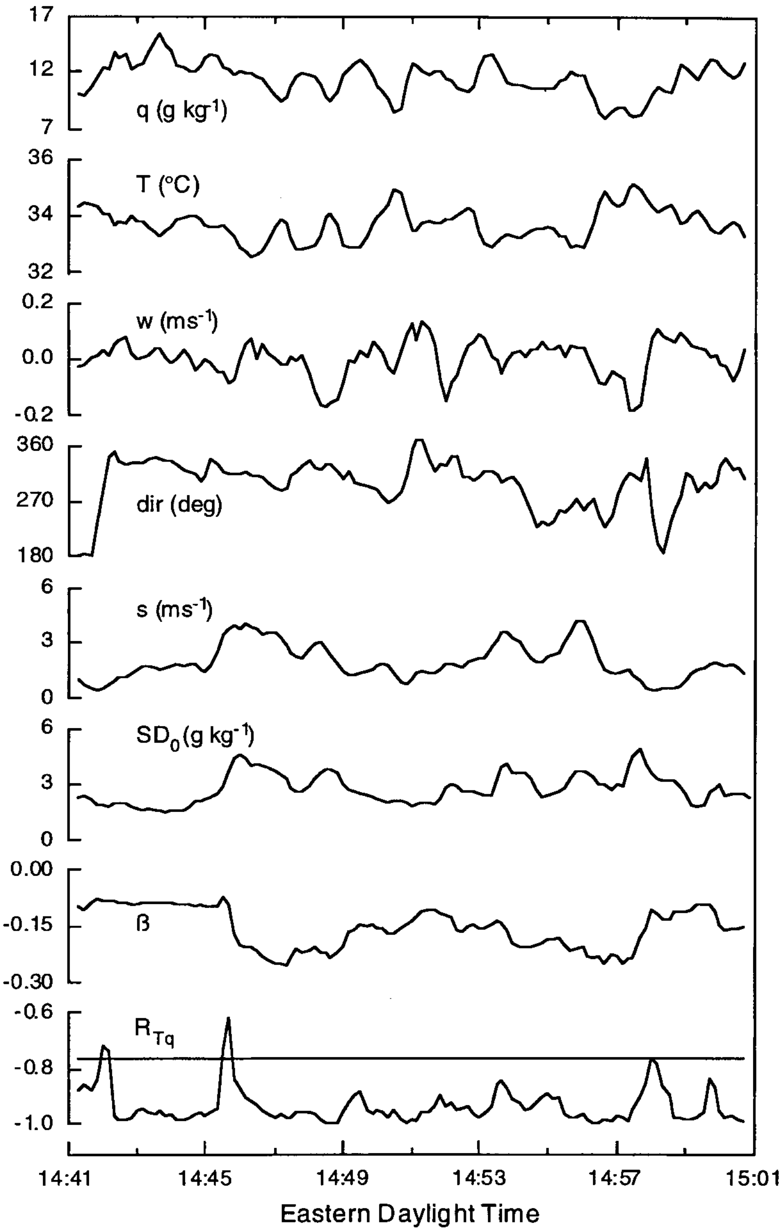


*Figure 2.* Time series of some meteorological quantities measured over paddy rice at Warrawidgee on February 5, 1997. The top six curves represent 30 s running averages of specific humidity $q$, air temperature $T$, vertical wind $w$, wind direction and horizontal wind speed, $s$, all at 2.4 m above water level, and saturation deficit, $SD$, measured within the rice canopy at 50 cm above water level. The curve $\beta$ shows the Bowen ratios calculated from the slope of the regression relationship between $T$ and $q$, each regression using 30 seconds (300 points) of raw data. The $R_{Tq}$ values shown were calculated for the same brief intervals. The horizontal line represents $R_{Tq}$ for the whole period. Fetch over the rice was 460 $\pm$ 30 m, depending on the wind direction. Net Radiation was steady within 1.5%.

To summarize our interpretation of these data, we seem to have observed a situation where upwind convection produced slow fluctuations in external wind speed and variable forcing of the IBL over the rice crop. This produced slow fluctuations in saturation deficit within the crop canopy, causing fluctuations in the local evaporation rate which were positively correlated with wind speed. As a result, periods of largest moisture flux were correlated with periods of most efficient transport (largest ratio of flux to gradient, or short-term diffusivity) so we expect the ratio of the average moisture flux to average humidity gradient – the average diffusivity – to be somewhat larger than predicted by Monin-Obukhov theory based on mean quantities. For the sensible heat flux, varying in complementary fashion to the latent heat flux, the situation is more complex since this flux is the residual term of the energy balance and could even change sign with variation in saturation deficit. Whatever the case, we no longer expect to find equal eddy diffusivities for $T$ and $q$. We develop this idea below.

## 3. Theory

Our theory deals with observations made about a reference level located within the fully-adjusted part of a stable layer at the base of a developing IBL. We do not deal specifically with our experimental situation at Warrawidgee since early analysis of the experimental results from there (unpublished) shows evidence that the lowest instrument level used was not within the fully adjusted layer, even though the fetch–height ratio used was larger than in most other experiments of this type. Our theory relates to an ideal situation: essentially it is the limiting case as $z \to 0$. We also assume that available energy is constant. Our aim is to show that unsteadiness of the wind can cause inequality of $K_T$ and $K_q$ even in this ideal situation.

### 3.1. Selection of Convenient Variables

Temperature and humidity are not convenient variables to work with because they are doubly linked at the ground: by the surface energy balance which forces variations in sensible and latent heat fluxes to be complementary, and by the transpiration process which links evaporation to the saturation deficit, which depends on both temperature and humidity. Because of this, any discussion of the generation of sensible heat flux is automatically linked to a discussion of the generation of latent heat flux, and vice versa.

It is possible to transform this coupled problem of flux generation into a pair of uncoupled problems by working with an alternative pair of scalar variables: the total heat (sensible plus latent) density, $\alpha$, and a linearized form of the saturation deficit, $d$ – rather than temperature, $T$, and humidity, $q$, directly. That is, we can link the fluxes of $\alpha$ and $d$ to the respective densities $\alpha$ and $d$ at the ground, $\alpha_0$ and $d_0$, each having no dependence on the other scalar at the ground. Results for $T$ and

$q$ can be reconstructed later. This procedure was first used by McNaughton (1976); it is outlined in Appendix B.

Our scalar variables, $\alpha$ and $d$, are defined by

$$\alpha = \rho c_p T + \rho\lambda q \tag{1}$$

and

$$d = \rho\lambda[q^*(\overline{T_0}) + s(\overline{T_0})(T - \overline{T_0}) - q]. \tag{2}$$

Symbols are conventional and are defined in Appendix B. The subscript '0' refers to the value at the surface. At the lower boundary the flux of $\alpha$, $F_\alpha$, is described by

$$F_{\alpha,0} = R_n - G \tag{3}$$

where the available energy, $R_n - G$ is taken to be steady during a measurement period of, say, 20 minutes. The lower boundary condition for $d$ is a relationship between the flux of saturation deficit, $F_{d,0}$, and the absolute concentration of saturation deficit

$$F_{d,0} = \varepsilon(R_n - G) - (1 + \varepsilon)\frac{d_0}{r_c} \tag{4}$$

where $\varepsilon$ is a parameter which depends only on mean surface temperature as defined in Appendix B, and $r_c$ is the canopy resistance of the vegetation.

### 3.2. Active and Inactive Fluctuations

Some time ago Townsend (1961) noted that turbulence intensities could vary widely within boundary layers having the same shear stress and velocity profiles. To account for this he proposed that "the motion at any point consists of two components, an active component determined by the stress distribution and an inactive component which does not transfer momentum or interact with the universal component". This is now referred to as Townsend's hypothesis. It has been supported in the work of Bradshaw (1967) and Katul et al. (1996). Our analysis makes literal use of Townsend's hypothesis and we extend its associated terminology of active and inactive wind components to scalar fluctuations by dividing them into active and inactive parts according to whether or not the components are correlated with vertical wind fluctuations. Since scalar fields add linearly, decomposition of scalar fluctuations in this way is allowable, regardless of whether Townsend's hypothesis is justified for (non-linear) velocity components. Thus active components of both velocity and scalar fluctuations are associated with eddy fluxes to or from the surface while inactive components are not.

In our work we associate 'active' fluctuations with the characteristic time scale $z/u_*$, and 'inactive' fluctuations with the much longer time scale characteristic of convective processes in the whole convective atmospheric boundary layer. A clean separation of a measured time series into active and inactive components would require strict spectral separation of the active and inactive fluctuations. This could be achieved, in principle, by setting the measurement level low enough to the ground since the surface time scale decreases as the ground is approached while the time scale for the external process does not. In practice such neatly divided spectra are rarely observed. For simplicity, we deal with the ideal case and suppose that fluctuations can be separated into active and inactive components by applying suitable filters to the data.

In accordance with convention we use Reynolds' notation of overbar and prime to indicate mean and deviation, but we extend this by using subscripts '$a$' and '$p$' to label the active and inactive parts of the deviations, respectively. (We avoid the subscript '$i$' to obviate confusion with Cartesian components of vectors. For mnemonic assistance, '*p*assive' is a synonym for inactive in ordinary speech, though the former has established a distinct technical meaning in micrometeorology). As usual, the mean cross-wind and vertical velocities are zero. We set the inactive components of vertical velocity to zero since inactive fluctuations are associated with larger eddies that move horizontally near the ground. Thus

$$u = \bar{u} + u'_p + u'_a \tag{5}$$

$$v = v'_p + v'_a \tag{6}$$

and

$$w = w'_a. \tag{7}$$

Scalars also have active and inactive components, so we write

$$\alpha = \bar{\alpha} + \alpha'_p + \alpha'_a \tag{8}$$

and

$$d = \bar{d} + d'_p + d'_a. \tag{9}$$

Next we assume that a decomposition operator is available that can 'filter out' the active fluctuations and comply with Reynolds' criteria for a well-behaved operator. We will call this a 'brief-average' operator to make the development more intuitive, though a more rigorous treatment would define it in terms of its properties rather than its practical specification. We base this assumption on the intuitive notion of existence of a spectral gap between active and passive components, but note that this does not preclude a more general justification for our methods.

Using the double overbar to denote 'brief-averaging' we write, by definition,

$$\overline{\overline{u'_a}} = \overline{\overline{v'_a}} = \overline{\overline{w'_a}} = \overline{\overline{\alpha'_a}} = \overline{\overline{d'_a}} = 0. \tag{10}$$

This brief-averaging operator has no effect on inactive fluctuations, so we also write

$$\overline{\overline{u'_p}} = u'_p; \quad \overline{\overline{\alpha'_p}} = \alpha'_p; \quad \text{etc.} \tag{11}$$

This is in addition to the whole-period averaging operator we have used above, which averages over a sufficiently long period that all fluctuations average to zero. That is

$$\overline{u'_p} = \overline{v'_p} = \overline{w'_p} = \overline{\alpha'_p} = \overline{d'_p} = 0. \tag{12}$$

Since the brief-averaging period is assumed ‘long’ with respect to $z/u_*$, we can write

$$\frac{\tau}{\rho} = -\overline{\overline{u'_a w'_a}}; \quad F_\alpha = \overline{\overline{w'_a \alpha'_a}}; \quad F_d = \overline{\overline{w'_a d'_a}} \tag{13}$$

where $\tau$ is the momentum flux and $\rho$ the density of air.

Consistent with our assumption that the active fluctuations and fluxes are in continuous equilibrium with the surface, we specify that the flux at our observation level equals the surface flux without bias or significant statistical fluctuation over a brief-averaging interval. We omit double overbars on the brief-averaged fluxes because we have no use for fluxes averaged over shorter periods: our brief period is the shortest time over which the fluxes at reference level can be matched up with conditions at the lower boundary.

This idea of a running equilibrium near the surface implies that, for example, the log-law holds during a brief interval, with a brief-averaged friction velocity defined by

$$u_*^2 = \frac{|\tau|}{\rho} = C_d(\bar{u} + s'_p)^2 \tag{14}$$

where $C_d = k^2/\ln^2(z/z_0)$ is the drag coefficient, $k$ is the von Kármán constant and $(\bar{u} + s'_p)$ is the mean wind speed over the brief period, $\sqrt{(\bar{u} + u'_p)^2 + v_p'^2}$. Equation (14) acknowledges that the brief-mean shear stress is not necessarily in the direction of the long-mean wind.

The absence of vertical flux divergence also allows us to apply (4) to the flux at an observation height a little way above the surface. Splitting $d_0$ into a mean part plus a inactive fluctuation then gives

$$F_d = \varepsilon(R_n - G) - (1 + \varepsilon)\frac{\bar{d}_0 + d'_{0,p}}{r_c}. \tag{15}$$

Like (14), this equation describes a flux which varies in step with inactive fluctuations in a driving variable, here $d_0$. As a surface value, $d_0$ is not easily measured.

To eliminate it we use the familiar resistance form of the transfer relationship, assuming again that there is no flux divergence between ground and observation level during a brief-averaging period. This can be written as

$$F_d = \frac{\left(1 + \frac{s'_p}{\bar{u}}\right)}{r_a}(\bar{d}_0 + d'_{0,p} - \bar{d} - d'_p) \tag{16}$$

where $r_a$ is the aerodynamic resistance for (all) scalars calculated from the mean wind speed, $\bar{u}$, and $s'_p$ and $\bar{u}$, $\bar{d}$ and $d'_p$ are all measured at reference height. The factor $(1 + s'_p/\bar{u})$ allows the aerodynamic resistance to vary in step with the slow (inactive) fluctuations in wind speed. We now eliminate $\overline{d_0} + d'_{0,p}$ between (15) and (16) to obtain an expression for $F_d$ in terms of quantities at reference-height, thus

$$F_d = \frac{(1+\varepsilon)}{r_c}\frac{\left[\frac{\varepsilon(R_n - G)r_c}{(1+\varepsilon)} - \bar{d} - d'_p\right]}{\left[1 + \frac{(1+\varepsilon)}{r_c}\frac{r_a}{\left(1 + \frac{s'_p}{\bar{u}}\right)}\right]}. \tag{17}$$

Here the first term within the upper square brackets is known as the equilibrium saturation deficit. For compactness we will write the difference between the observed mean saturation deficit and the equilibrium saturation deficit as $\overline{\Delta d}$, and so, after a little re-arrangement, write (17) as

$$F_d = -\frac{\overline{\Delta d} + d'_p}{\left[\frac{r_c}{(1+\varepsilon)} + \frac{\bar{u}r_a}{(\bar{u} + s'_p)}\right]}. \tag{18}$$

This equation shows that $F_d$ is affected by the inactive variations in both wind speed and saturation deficit at a reference height close above the ground.

### 3.3. Eddy diffusivities for total energy and saturation deficit

The eddy diffusivity of a scalar is defined as the ratio of the average flux of a scalar to the average gradient of that scalar. Over a whole observation period this gives

$$K_\alpha = -\frac{\overline{F_\alpha}}{\overline{\frac{\partial\alpha}{\partial z}}} \tag{19}$$

and

$$K_d = -\frac{\overline{F_d}}{\overline{\dfrac{\partial d}{\partial z}}}. \tag{20}$$

We assume that the total energy flux is described by the usual surface-layer relationships during a brief-averaging period so, ignoring stability effects, which should be small close to the ground, we have

$$\overline{\overline{\frac{\partial \alpha}{\partial z}}} = -\frac{F_\alpha}{kzu_*}. \tag{21}$$

In this equation $z$ is fixed for a given observation level and $F_\alpha$ is independent of $u_*$. Taking an overall mean yields

$$\overline{\frac{\partial \alpha}{\partial z}} = -\frac{\overline{F_\alpha}}{kz}\overline{\left(\frac{1}{u_*}\right)}. \tag{22}$$

The mean reciprocal friction velocity is given by

$$\overline{\left(\frac{1}{u_*}\right)} = \frac{1}{\sqrt{C_d}\bar{u}}\overline{\left(\frac{1}{1+\dfrac{s'_p}{\bar{u}}}\right)} \tag{23}$$

using (14). Expanding the term $(1 + s'_p/\bar{u})^{-1}$ in this equation to a series in $s'_p/\bar{u}$, taking the whole-period average and combining the result with (22) gives

$$\overline{\frac{\partial \alpha}{\partial z}} = -\frac{\overline{F_\alpha}}{kz\sqrt{C_d}\bar{u}}\left(1 + \frac{\overline{s'^2_p}}{\bar{u}^2} - \frac{\overline{s'^3_p}}{\bar{u}^3} + \cdots\right) \tag{24}$$

so the eddy diffusivity for $\alpha$ will be given by

$$K_\alpha = kz\sqrt{C_d}\bar{u}\Bigg/\left(1 + \frac{\overline{s'^2_p}}{\bar{u}^2} - \frac{\overline{s'^3_p}}{\bar{u}^3} + \cdots\right). \tag{25}$$

We see that averaging (21) over the inactive wind fluctuations tends to emphasize periods of smaller wind speed and larger gradients, so causing $K_\alpha$ to be smaller than predicted by Monin-Obukhov theory, where $K_\alpha = kz\sqrt{C_d}\bar{u}$. The size of the reduction depends principally on the size of $\overline{s'^2_p}/\bar{u}^2$. If the standard deviation of inactive wind speed is, say, 20% of the mean wind velocity then $K_\alpha$ will be just 4% less than the Monin-Obukhov value $kz\sqrt{C_d}\bar{u}$ . (This change closely parallels the 4% change that Bradshaw (1967) calculated for the apparent value of the von Kármán constant in the log wind profile equation when $\overline{u_p'^2}/\bar{u}^2 = 0.04$.) A reduction of 4% in $K_\alpha$ (or $k$) would be hard to detect experimentally, but conditions with very light and variable winds might well produce larger values of $\overline{s_p'^2}/\bar{u}^2$ with easily detectable effects on $K_\alpha$.

We now turn to the effective diffusivity for saturation deficit, using (20). Things are more complex here because $F_d$ is linked to wind speed through the effect of wind variations on $d_0$. We must find expressions for both $\overline{F_d}$ and $\overline{\partial d/\partial z}$ with this in mind.

Addressing the mean gradient first, we begin again with the equilibrium equation for the brief-period diffusivity and write

$$\frac{\overline{\overline{\partial d}}}{\partial z} = -\frac{F_d}{kzu_*} \tag{26}$$

where, this time, we must substitute for $F_d$, using (18), and $u_*$, using (14), to get

$$\frac{\overline{\overline{\partial d}}}{\partial z} = \frac{\overline{\Delta d} + d_p'}{kz\sqrt{C_d}(\bar{u} + s_p')\left[\dfrac{r_c}{(1+\varepsilon)} + \dfrac{\bar{u}r_a}{(\bar{u} + s_p')}\right]} \tag{27}$$

or

$$\frac{\overline{\overline{\partial d}}}{\partial z} = \frac{\overline{\Delta d} + d_p'}{kz\sqrt{C_d}\bar{u}r_d\left[1 + \eta\dfrac{s_p'}{\bar{u}}\right]} \tag{28}$$

where $\eta = r_c/[(1+\epsilon)r_a + r_c]$ and $r_d = r_c/(1+\epsilon) + r_a$. Here $\eta$ is a coupling coefficient and is complementary to the decoupling coefficient, $\Omega(= 1 - \eta)$ introduced by McNaughton and Jarvis (1983). The reciprocal of the term in square brackets can be expanded into a Taylor series in $s_p'/\bar{u}$ to give

$$\begin{aligned}\frac{\overline{\overline{\partial d}}}{\partial z} &= \frac{\overline{\Delta d}}{kz\sqrt{C_d}r_d\bar{u}}\left(1 + \frac{d_p'}{\overline{\Delta d}}\right)\\ &\quad\times\left(1 - \frac{\eta s_p'}{\bar{u}} + \left(\frac{\eta s_p'}{\bar{u}}\right)^2 - \left(\frac{\eta s_p'}{\bar{u}}\right)^3 + \cdots\right)\end{aligned} \tag{29}$$

Multiplying out (29) and taking the whole-period mean then leads to

$$\frac{\overline{\partial d}}{\partial z} = \frac{\overline{\Delta d}[1 - \eta S]}{kz\sqrt{C_d}r_d\bar{u}} \tag{30}$$

where $S$ is the series

$$S = \left( \frac{\overline{s'_p d'_p}}{\bar{u}\overline{\Delta d}} - \eta \frac{\overline{s'^2_p}}{\bar{u}^2} - \eta \frac{\overline{s'^2_p d'_p}}{\bar{u}^2 \overline{\Delta d}} + \eta^2 \frac{\overline{s'^3_p}}{\bar{u}^3} \cdots \right). \tag{31}$$

An expression for the mean flux is derived in a similar way by averaging (18). The result is

$$\overline{F_d} = -\frac{\overline{\Delta d}}{r_d}[1 + (1 - \eta)S]. \tag{32}$$

The effective diffusivity for saturation deficit is then given by

$$K_d = \frac{kz\sqrt{C_d}\bar{u}[1 + (1 - \eta)S]}{[1 - \eta S]}. \tag{33}$$

This diffusivity also deviates from that predicted by standard Monin-Obukhov theory, but this time the diffusivity is increased. The amount of increase depends on the value of $\eta$ and the various moments of $s'_p$ and $d'_p$. We can estimate the size of this increase by finding typical values for $\eta$ and $S$.

Over a lush irrigated cereal crop at moderate wind speed a typical value for $\eta$ is 0.3. The leading term of $S$ is $\overline{s'_p d'_p}/\bar{u}\overline{\Delta d}$. To estimate it we note that the positive correlation between $s'_p$ and $d'_p$ may well be very high because in advective situations it is the large-scale variations in wind speed that directly cause the slow variations in $d$ by changing the horizontal gradient of $d$ across a field. Thus we expect $\overline{s'_p d'_p} \approx \sigma_{s,p}\sigma_{d,p}$, where the sigmas indicate the standard deviations of the subscript variables. [Note: the data in Figure 2 might suggest a smaller correlation, but these data are limited by imperfect separation of active and inactive components of wind speed by the simple filtering method used.] As in earlier examples, we let $\sigma_{s,p}/\bar{u}$ be about 0.2. The value of $\sigma_{d,p}/\overline{\Delta d}$ is harder to estimate *a priori*. Supposing that it is about 0.5, we find that $\overline{s'_p d'_p}/\bar{u}\overline{\Delta d}$ is of order 0.1. Using the same estimates we find the next term is about -0.01. The third and higher order moments should be smaller and we ignore them for the present purpose. This means that $S \approx 0.11$ and $K_d$ is about 8% larger than the value found from Monin-Obukhov similarity theory applied to the whole-period averaged variables.

Finally, we can write an equation for the ratio of the diffusivities by combining (25) and (33). It is

$$\frac{K_\alpha}{K_d} = \frac{[1 - \eta S]}{\left[1 + \frac{\overline{s'^2_p}}{\bar{u}^2} - \frac{\overline{s'^3_p}}{\bar{u}^3}\right][1 + (1 - \eta)S]} \tag{34}$$

Clearly $K_\alpha$ will not equal $K_d$ except in special cases. One such case arises when there are no inactive fluctuations in wind speed, so that $s'_p$ values and $S$ are zero.

This reflects our assumption that scalar diffusivities are equal in steady conditions. Another case arises when $r_c$ is very large, so that $\eta \to 1$ and $\overline{\Delta d} \to \infty$. The term in the denominator multiplied by $(1-\eta)$ then vanishes, as do the first, third, etc. terms in $S$, defined in (31), so that the second, fourth etc., terms match their denominator counterparts and the terms cancel; transport similarity is restored even though neither diffusivity fits Monin-Obukhov theory exactly. This similarity is reassuring because the boundary condition (4) for $F_d$ becomes a constant flux boundary condition when $r_c \to \infty$, and so becomes similar to the constant-flux boundary condition for $F_\alpha$; the two scalars are released similarly so they are transported similarly.

In other cases the ratio $K_\alpha/K_d \neq 1$. In our previous examples we found $K_\alpha$ to be about 4% less than that predicted by Monin-Obukhov theory and $K_d$ to be about 8% more, so combining these estimates we have $K_\alpha/K_d \approx 0.88$. This is quite significantly different from one. Of course, the exact value depends on our assumed values. In particular, it is sensitive to the value of $\sigma_{s,p}/\bar{u}$; this could be rather larger than 0.2 in light winds with strong convection in the planetary boundary layer over a dry upwind plane, so larger departures from similarity are quite possible.

In summary, our analysis shows that total energy and saturation deficit are not transported similarly at the base of an advective inversion over well-watered vegetation when that IBL is influenced by fluctuations in the larger-scale wind field. In light, variable winds this non-similarity can, it seems, become quite marked.

### 3.4. The Eddy Diffusivities for Temperature and Humidity

The ratio of the effective eddy diffusivities for temperature and humidity, $K_T/K_q$, can be found from the ratio $K_\alpha/K_d$. We begin by writing out the basic definitions of $K_\alpha$ and $K_d$ in terms of temperature and humidity, thus

$$K_\alpha = -\frac{\rho c_p \overline{w'T'} + \rho\lambda\overline{w'q'}}{\rho c_p \dfrac{\partial \bar{T}}{\partial z} + \rho\lambda \dfrac{\partial \bar{q}}{\partial z}} \tag{35}$$

and

$$K_d = -\frac{\varepsilon\rho c_p \overline{w'T'} - \rho\lambda\overline{w'q'}}{\varepsilon\rho c_p \dfrac{\partial \bar{T}}{\partial z} - \rho\lambda \dfrac{\partial \bar{q}}{\partial z}}. \tag{36}$$

We must solve these two equations for the quantities $-\overline{w'T'}/(\partial\bar{T}/\partial z)$ and $-\overline{w'q'}/(\partial\bar{q}/\partial z)$, which are $K_T$ and $K_q$, respectively. On defining $\beta_g$ as the gradient Bowen Ratio

$$\beta_g = \frac{c_p \dfrac{\partial \bar{T}}{\partial z}}{\lambda \dfrac{\partial \bar{q}}{\partial z}} \tag{37}$$

we find

$$K_T = \frac{(K_\alpha + \varepsilon K_d)}{(\varepsilon + 1)} + \frac{(K_\alpha - K_d)}{\beta_g(\varepsilon + 1)} \tag{38}$$

and

$$K_q = \frac{(\varepsilon K_\alpha + K_d)}{(\varepsilon + 1)} - \frac{\beta_g \varepsilon (K_d - K_\alpha)}{(\varepsilon + 1)}. \tag{39}$$

The ratio $K_T/K_q$ then can be written as

$$\frac{K_T}{K_q} = \frac{\xi + \varepsilon + \beta_g^{-1}(\xi - 1)}{\varepsilon\xi + 1 + \beta_g \varepsilon(\xi - 1)} \tag{40}$$

where $\xi = K_\alpha/K_d$. This shows that $K_T$ and $K_q$ are equal in only three cases: when $\xi = 1$ (i.e. $K_\alpha = K_d$), when $\beta_g = \varepsilon^{-1}$ (so $F_d$ = 0, from $K_T = K_q$ and (B9)), and when $\beta_g = -1$ (so $F_\alpha = 0$, from $K_T = K_q$ and (B2)). Otherwise $K_T$ and $K_q$ are not equal. $K_T/K_q$ is shown plotted against $\beta_g$ in Figure 3.

Notice that $K_T/K_q$ has an important singularity where $\beta_g = 0$. This derives from the singularity in $K_T$ when $\beta_g = 0$ because then the temperature gradient is zero even though the heat flux can be non-zero. Near this singularity the ratio $K_T/K_q$ can depart far from unity, even when $K_\alpha/K_d$ is only slightly different from one. Gradient Bowen ratios in the range $-0.2 < \beta_g < 0.1$ occur often in advective situations. A second singularity formally exists at very large $\beta_g$ but it is of no practical interest.

## 4. Discussion

The theoretical results from the previous section agree with the experimental results from Mead in two respects: they find $K_T/K_q > 1$ near the base of an advective inversion over an irrigated area, in agreement with our theory; and they find a relationship between $K_T/K_q$ and $\Delta T/\Delta q$, with $K_T/K_q$ departing most from unity as $\Delta T/\Delta q \rightarrow 0$ (Figure 3), as predicted by our theory. However, quantitative agreement with our theory would require $K_\alpha/K_d \approx 0.6$. This value is smaller than we think likely for their situation. Overall, we consider the interpretation of the results from Mead still to be an open question.

Non-similarity of the diffusivities can cause errors when calculating the sensible heat flux by the Bowen ratio method. In these calculations it is usual to assume $K_T = K_q$ and to use the equation

$$H_g = \frac{\beta_g}{1 + \beta_g}(R_n - G) \tag{41}$$

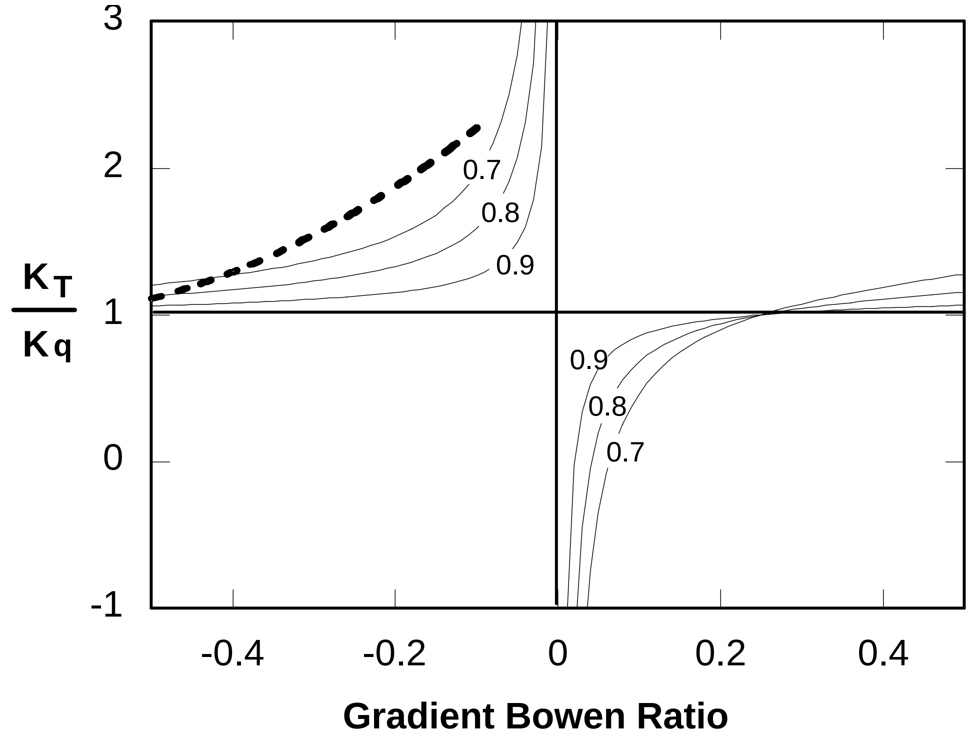


*Figure 3.* Diffusivity ratio $K_T/K_q$ plotted against the gradient Bowen ratio, $\beta_g$, for $\varepsilon$ = 4 and three values of the ratio $K_\alpha/K_d$. Notice that there is a singularity at $\beta_g$ = 0 and that $K_T/K_q = 1$ at $\beta_g = 1/\varepsilon$ and $\beta_g = -1$. Also plotted is the relationship between $K_T/K_q$ and $\beta_g$ found by Verma et al. (1978), averaged by eye over all their experiments (dashed line).

rather than the strictly correct equation

$$H = \frac{\frac{K_T}{K_q}\beta_g}{1 + \frac{K_T}{K_q}\beta_g}(R_n - G). \tag{42}$$

With these and the diffusivity ratio from (40) we can calculate the error as a fraction of available energy. Calculations show that there is no singularity in the calculated value of the sensible heat flux when $\beta_g = 0$, and that errors are a few percent or less of available energy in unstable and near-neutral conditions when $\xi$ = 0.8. For strongly stable conditions $\beta_g \rightarrow -1$ and percentage errors become significant, especially for $\beta_g < -0.2$. That is, Bowen ratio measurements of the surface energy fluxes will usually be satisfactory, except within quite intense inversions. A greater problem is to locate the instruments close enough to the ground to avoid errors from flux divergence.

## 5. Conclusion

It appears that there are two mechanisms that can lead to non-similarity in the eddy diffusivities for heat and water vapour near the base of an internal boundary layer over an irrigated crop downwind of a dry area. The first is lack of full adjustment of the turbulent transport processes to the new surface and the second is variations in the external wind with accompanying variations in the fluxes of heat and vapour from the field. The first of these presents no surprise, except that in an IBL that is stable at its base the fetch-to-height requirement for similarity of diffusivities must be a great deal larger than is generally recognised. The second derives from unsteadiness of the external wind – a common phenomenon which has been little studied. In intuitive terms, non-similarity arises because periods of higher wind speed are associated with larger saturation deficit and larger evaporation rate. Thus, for saturation deficit, periods of largest flux coincide with periods of most efficient transport and relatively small gradients. For total energy there is no such correlation because the total energy flux is steady at its source. As a result the eddy diffusivity for saturation deficit, calculated as the ratio of average flux to average gradient, will appear larger than that for total energy calculated in a similar way. This is reflected in the ratio of diffusivities for temperature and humidity, where the ratio $K_T/K_q$ can depart greatly from one if the Bowen ratio is small. Despite this, assuming $K_T = K_q$ and using the Bowen ratio method for practical calculations of the surface energy fluxes will usually incur only minor error.

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## Appendix A: Non-Similarity of Diffusivities within an Advective Inversion Formed Downwind of a Dry-to-Wet Transition

Consider an advective situation where the heat and vapour fluxes from a dry upwind area are marked by a yellow tracer and the fluxes from a wetter downwind area are marked by a red tracer. Since scalar fields add linearly, the sensible and latent heat fluxes at any point over the wetter area can be decomposed into red and yellow components. Thus

$$H' = H_Y + H_R \tag{A1}$$

and

$$E' = E_Y + E_R. \tag{A2}$$

Because the source characteristics upwind are identical, transport of the upwind-released scalars will be identical and will share the same diffusivity everywhere. The same can be said of the red-traced scalars released downwind. Therefore diffusion equations for these fluxes can be written thus:

$$-\rho c_p K_T \frac{\partial T}{\partial z} = -\rho c_p K_Y \frac{\partial T_Y}{\partial z} - \rho c_p K_R \frac{\partial T_R}{\partial z} \tag{A3}$$

and

$$-\rho \lambda K_q \frac{\partial q}{\partial z} = -\rho \lambda K_Y \frac{\partial q_Y}{\partial z} - \rho \lambda K_R \frac{\partial q_R}{\partial z} \tag{A4}$$

where $K_Y$ and $K_R$ are the effective diffusivities associated with the yellow-traced and red-traced scalars, respectively, and $K_T$ and $K_q$ are the diffusivities associated with the total sensible heat and vapour fluxes, respectively. Taking the ratio of these we can write

$$\frac{\rho c_p K_T \dfrac{\partial T}{\partial z}}{\rho \lambda K_q \dfrac{\partial q}{\partial z}} = \frac{\rho c_p K_Y \dfrac{\partial T_Y}{\partial z} + \rho c_p K_R \dfrac{\partial T_R}{\partial z}}{\rho \lambda K_Y \dfrac{\partial q_Y}{\partial z} + \rho \lambda K_R \dfrac{\partial q_R}{\partial z}}. \tag{A5}$$

If the upwind area is quite dry then the ‘yellow’ humidity gradient will be zero and the ‘red’ humidity gradient will equal the total humidity gradient, which then cancels in the equation. Also, the total temperature gradient can be decomposed into its yellow and red parts, so we can reduce (A5) to

$$\frac{K_T}{K_q} = \frac{1 + \dfrac{K_Y}{K_R}\left(\dfrac{\partial T_Y}{\partial z} \Big/ \dfrac{\partial T_R}{\partial z}\right)}{1 + \left(\dfrac{\partial T_Y}{\partial z} \Big/ \dfrac{\partial T_R}{\partial z}\right)}. \tag{A6}$$

In this equation the ratio of the temperature gradients will be negative within an advective inversion. Also, we expect that $K_Y/K_R > 1$ because larger eddies will transport red scalars less effectively. This follows from the mechanism discussed by Lang et al. (1983) and Kroon and Bink (1996) whereby the larger downsweeps are

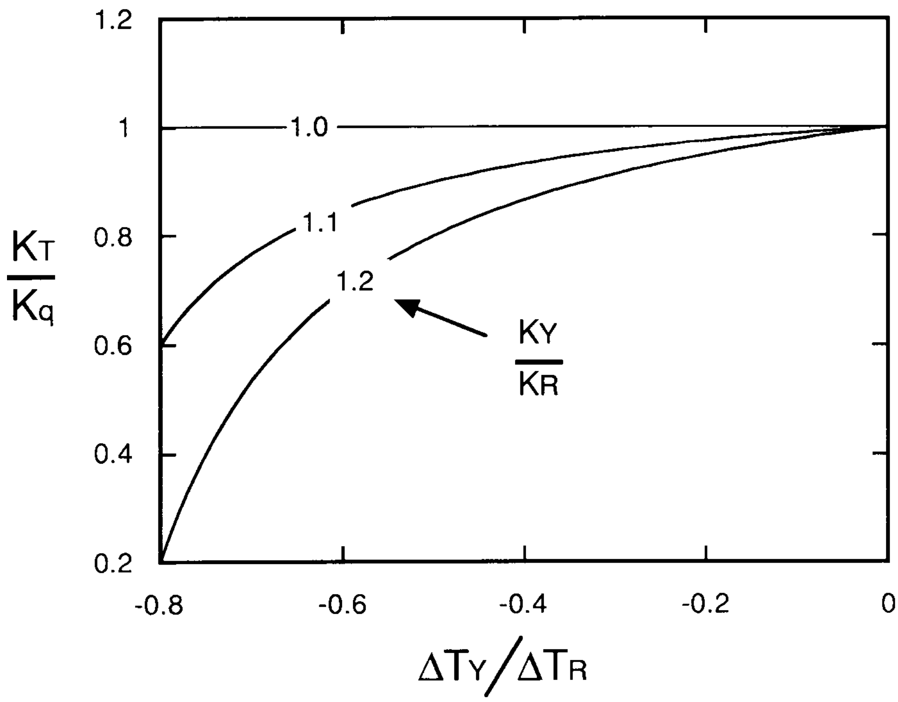


*Figure A1.* Ratio of diffusivities for temperature and humidity plotted against the ratio of the upwind-sourced and downwind-sourced parts of the temperature gradient. The values plotted are all negative, indicating that the gradients are in opposite directions so, since the advected upwind component is necessarily lapse, the total downwind gradient is inversion. Calculations are made using Equation (A6).

less depleted in red scalar than would be predicted were the gradients at observation level extrapolated upwards using the equilibrium profile forms associated with the Monin-Obukhov similarity framework. The transport effectiveness of downsweeps will not be so limited for the yellow scalar. The ratio $K_T/K_q$ plotted against the ratio of the 'yellow' and 'red' temperature gradients is shown in Fig. A1. It can be observed that $K_T/K_q$ is always less than one within an advective inversion that forms downwind of a dry-to-wet transition.

## Appendix B: Selection of Scalar Variables to Achieve Independent Lower Boundary Conditions

The following is a more explanatory account of the material presented by McNaughton (1976). Here we deal only with the case where available energy is constant.

The energy fluxes from a vegetated surface are constrained by two boundary conditions: the first is that energy must be conserved at the ground, while the second is a canopy model that describes how evaporation responds to environmental conditions. Taking the energy balance first, we have

$$R_n - G = H_0 + \lambda E_0 \tag{B1}$$

where $H$, $\lambda E$, $R_n$ and $G$ are the sensible heat, latent heat, net radiation and ground heat flux densities, respectively and the subscript "0" indicates the surface values. These quantities are well defined over any interval at the surface, though eddy-covariance measurements of convective fluxes made above the surface usually require averaging over time to smooth the statistical fluctuations caused by the turbulence. Therefore boundary conditions are usually written in time-averaged form. Taking the fluxes a little way above the surface and using overbars to indicate an average over a period of, say, 20 minutes, we can write the sum of the turbulent fluxes as

$$\bar{H} + \lambda \bar{E} = \rho c_p \overline{w'T'} + \rho\lambda \overline{w'q'} \tag{B2}$$

or

$$\overline{F_\alpha} = \overline{w'\alpha'} \tag{B3}$$

where the scalar

$$\alpha = \rho c_p T + \rho\lambda q \tag{B4}$$

is the total heat density with units of J m$^{-3}$ and $F_\alpha$ is the total convective heat flux with units of W m$^{-2}$. Defined thus, $\alpha$ is $\rho c_p$ times the equivalent temperature; we include $\rho c_p$ in the definition here to keep our equations tidier. Total heat is a conserved scalar and is transported in exactly the same way as any other conserved scalar, such as temperature, virtual temperature or refractive index. We will use only fluctuations or gradients of $\alpha$ in our equations, so it is unimportant where we place the zero point of the scale for total heat density.

In this paper we assume that the total convective heat flux is insensitive to fluctuations in either surface temperature or humidity, so the surface fluxes $H_0$ and $\lambda E_0$ vary in a truly complementary fashion within an averaging period. This is a good approximation for dense vegetation which shades the ground well. This being the case, $F_{\alpha,0}$ can be known from measurements of $R_n$ and $G$, without reference to the concentrations of any other scalars; the lower boundary condition (B1) can therefore be written as $F_{\alpha,0} = (R_n - G)$. Timofeev (1954) was the first to use $\alpha$ as a variable for this purpose.

The second boundary condition is the vegetation model which describes evaporation at the surface; for this we use the 'big-leaf' canopy equation:

$$\lambda E_0 = \frac{\rho\lambda[q^*(T_0) - q_0]}{r_c}. \tag{B5}$$

Here $[q^*(T_0) - q_0]$ is the surface value of the saturation deficit and $r_c$ is the canopy resistance. For our second composite variable we define $d$ by

$$d = \rho\lambda[q^*(\overline{T_)}) + s(\overline{T_0})(T - \overline{T_0}) - q] \tag{B6}$$

where $s\ (\overline{T_0})$ is the slope of the relationship between saturation specific humidity and temperature determined at mean surface temperature and prevailing surface pressure. We include the $\rho\lambda$ in the definition so units are J m$^{-3}$. The values for the parameters $s(\overline{T_0})$ and $\overline{T_0}$ in (B6) are selected for each period such that $d_0 = \rho\lambda[q^*(\overline{T_0}) + s(\overline{T_0})(T_0 - \overline{T_0}) - q_0]$ is a close approximation to the real saturation deficit at the surface, $\rho\lambda[q^*(T_0) - q_0]$. Defined in this linearized form, $d$ is a linear combination of the conserved scalars $T$ and $q$ so it too is a conserved scalar. Priestley and Taylor (1972) were the first to use $d$ in this way. We shall refer to $d$ as the 'saturation deficit' despite its departure from the true saturation deficit, especially far above the ground.

With $d$ so defined we can write the boundary condition (B5), to a very good approximation, as

$$\lambda E_0 = \frac{d_0}{r_c}. \tag{B7}$$

Though compact, this form of the boundary condition is still not convenient because it links the flux of one scalar – humidity – to the concentration of another – saturation deficit. We would rather link the flux of $d$ to $d_0$. To achieve this we begin by writing an equation for the average flux of saturation deficit, $\overline{F_d}$, as

$$\overline{F_d} = \overline{w'd'} \tag{B8}$$

which can be expanded using (B6) to give

$$\overline{F_d} = \varepsilon\rho c_p\overline{w'T'} - \rho\lambda\overline{w'q'} \tag{B9}$$

in which the parameter $\varepsilon(\overline{T_0})$ is the dimensionless ratio $s(\overline{T_0})\lambda/c_p$. We can identify the two terms on the right here with the sensible and latent heat fluxes. In general the flux defined in (B9) will equal the surface flux only over averaging times long enough to remove the statistical 'noise' introduced by turbulence. However, as we approach the surface the averaging interval can become much shorter (it scales on $z/u_*$), so at the surface we can drop the overbars and write

$$F_{d,0} = \varepsilon H_0 - \lambda E_0 \tag{B10}$$

in which $H_0$ can be eliminated, using (B1), to give

$$F_{d,0} = \varepsilon(R_n - G) - (1 + \varepsilon)\lambda E_0. \tag{B11}$$

Finally, $\lambda E_0$ can be substituted using (B7) to give a form of the lower boundary condition in which the flux of saturation deficit depends only on the saturation deficit itself, as required:

$$F_{d,0} = \varepsilon(R_n - G) - (1 + \varepsilon)\frac{d_0}{r_c}. \tag{B12}$$

The upshot of all this is that we now have two conserved scalars, $\alpha$ and $d$, which do not interact at the surface, but which are transported by the same turbulent flow. The surface flux of $\alpha$ is independent of variations in wind speed or saturation deficit, while the flux of $d$ will track any variations in $d_0$.

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